CREATIVE EDUCATION FOUNDATION, KARKALA

SECOND PU ANNUAL EXAMINATION APRIL-2023

BASIC MATHS DETAILED SOLUTION

PART - A

I. Answer all the TEN multiple choice questions: **(10 × 1 = 10)**

1. If $A = \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$ then the matrix 2A will be

a) $\begin{bmatrix} 2 & -6 \\ 4 & 8 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 4 \\ -6 & 8 \end{bmatrix}$ c) $\begin{bmatrix} 8 & -6 \\ 4 & 2 \end{bmatrix}$ d) $\begin{bmatrix} 2 & 4 \\ 1 & -3 \end{bmatrix}$

Ans.: $A = \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$; $2A = \begin{bmatrix} 2 & -6 \\ 4 & 8 \end{bmatrix}$

2. The value of $\begin{vmatrix} 3200 & 3201 \\ 3202 & 3203 \end{vmatrix}$ is

a) 4 b) 0 c) –2 d) 2

Ans.: $\begin{vmatrix} 3200 & 3201 \\ 3202 & 3203 \end{vmatrix} = -2$

3. How many different arrangements can be made with the letters of the word "MONDAY" ?

a) 24 b) ${}^6P_4$ c) 720 d) 6

Ans.: n = 6 total ways = 6!=720

4. In how many ways can 10 people be seated around a table ?

a) 10! b) 9! C) 8! D) 7!

Ans.: (10 – 1)! = 9!

5. Given p: 3x = 9, q : x < 7 then, symbolic from of "3x = 9 or x < 7" is

a) $p \vee q$ b) $\sim p \vee q$ c) $p \wedge q$ d) $p \vee \sim q$

Ans.: $p \vee q$

6. The duplicate ratio of 2 : 3 is

a) 8 : 27 b) 3 : 2 c) 4 : 9 d) 9 : 4

Ans.: Duplicate ratio = $2^2 : 3^2$ = 4 : 9.

7. If $\sin A = \frac{1}{2}$ then the value of cos 2A is

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$ d) $\frac{\sqrt{3}}{2}$

Ans.: $\sin A = \frac{1}{2} \Rightarrow A = 30^o$, $\cos 2A = \cos 60^o = \frac{1}{2}$

8. The centre of the circle $x^2 + y^2 - 4x - y - 5 = 0$ is

a) (2, 1) b) $\left(2, \frac{1}{2}\right)$ c) $\left(1, \frac{1}{2}\right)$ d) (1, 2)

Ans.: $2y = -4 \Rightarrow g = -2$, $2f = -1 \Rightarrow f = -\frac{1}{2}$. centre = $(2, \frac{1}{2})$

9. If $y = 5e^x - \log x - 3\sqrt{x}$ then $\frac{dy}{dx}$ is

a) $5e^x - \frac{1}{x} - \frac{3}{2\sqrt{x}}$ b) $5e^x - \frac{1}{x^2} - \frac{3\sqrt{x}}{2}$ c) $5e^x - x - \frac{3}{2\sqrt{x}}$ d) $5e^x - \frac{1}{x} - 3\sqrt{x}$

Ans.: $\frac{dy}{dx} = 5e^x - \frac{1}{x} - \frac{3}{2\sqrt{x}}$

10. The value of $\int \frac{5}{x} dx$ is

a) 5 log x + C b) $\frac{-5}{x^2} + C$ c) log x + C d) $\frac{1}{5}$ log x + C

Ans.: 5 log x + C

II. Fill in the blanks by choosing the appropriate answer from the brackets given below : (5×1= 5)

(35, 4500, 9, 5%, $\frac{19}{2}$)

11. If ${}^nC_4 = {}^nC_5$ then, the value of n is ……

Ans.: ${}^nC_4 = {}^nC_5 \Rightarrow$ n = 5 + 4 = 9.

12. The fourth proportional of 6, 14, 15 is ……

Ans.: $6 : 14 :: 15 : x \Rightarrow x = \frac{14 \times 15}{6} = 36$

13. The amount of stock that can be bought for Rs. 3,375 at Rs. 75 is ……..

Ans.: Stock purchased = $\frac{100 \times 3,375}{76} = 4,500$

1. 14. Rama paid Rs. 60 as sales tax on a Titan Rag watch worth Rs. 1,200 then the rate of sales tax is ……..…

Ans.: ST% = $\frac{ST}{M.V.} \times 100 = \frac{60}{1200} \times 100 = 5\%$

15. The value of $\lim_{x \to 4} \left(\frac{4x+3}{x-2}\right)$ is ……

Ans.: $\lim_{x \to 4}\left(\frac{4x+3}{x-2}\right)=\frac{19}{2}$

III. Answer all the following questions : **(5 × 1 = 5)**

16. Negate : $\sim p \to q$.

Ans.: $\sim(\sim p \to q) \equiv \sim p \wedge \sim q$.

17. A bill was drawn on 14 – 3 – 2013 for 3 months find the legally due date.

Ans.: Legally due date = Date of drawing + bill period + grace period

= 14 – 3 – 2013

0 – 3 – 0

3 – – –

17 – 6 – 2013

18. Define learning index.

Ans.: learning index $= \frac{\log(\text{learning effect})}{\log 2}$

19. If the length of the latus rectum of $x^2 = 4ky$ is 8, find the value of k.

Ans.: $x^2 = 4ky$

$4a = 4k$

$8 = 4k \Rightarrow k = \frac{8}{4} = 2$

20. If the total cost of an article is $C = x^2 + 5x + 7$ where x indicates quantity, find its marginal cost.

Ans.: $C = x^2 + 5x + 7$

M. C $= \frac{dy}{dx} = 2x + 5$

PART – B

IV Answer any nine questions : **(9 × 2 = 18)**

21. If $A=\begin{bmatrix}2 & -1\\ 1 & 4\end{bmatrix}$ and $B=\begin{bmatrix}-3 & 2\\ -1 & 4\end{bmatrix}$ find (AB)'.

Ans.: $AB=\begin{bmatrix}-5 & 0\\ -7 & 18\end{bmatrix}$ $(AB)'=\begin{bmatrix}-5 & -7\\ 0 & 18\end{bmatrix}$

22. In how many ways can 6 boys and 6 girls be arranged in a row so that

a) All girls are together

b) All boys are not together

Ans.: Boys = 6 Girls = 6

a) Treat 6 girls as single unit.

Total = 6B + 1 single unit = 7

7 people can be, arranged in 7! ways and followed by 6 girls can be arranged in 6! ways

Total = $7! \times 6!$

b) All boys not together :

Total = 12 No. of boys = 12!

Boys together can be arranged in $7! \times 6!$ Ways

$\therefore$ Not together = $12! - (7! \times 6!)$

23. Two fair coin are tossed simultaneously. Find the probability of

a) getting two heads

b) atleast one head

Ans.: S = {HH, HT, TH, TT}

A : getting two heads P(A) = $\frac{1}{4}$

B : getting atleast one head, P(B) = $\frac{3}{4}$

24. If the compound proposition $p \rightarrow (q \vee r)$ is false, then find the tr4ugth values of p, q and r.

Ans.: $p \rightarrow (q \vee r) \equiv F$

$p = T \;\&\; q \vee r \equiv F \Rightarrow q = F \;\; r = F$

$\therefore$ P = T, q = F, r = F

25. A ratio in the lowest terms is 3 : 7. If the difference between the quantities is 24. Find the quantities.

Ans.: Let the terms are 3x and 7x

Given : $7x - 3x = 24$

$4x = 24$

$x = 6$

$\therefore$ items are $3 \times 6 = 18$, $7 \times 6 = 42$.

26. Banker's Discount and Banker's Gain on a certain bill due after some time are Rs. 927 and Rs. 27 respectively, find the face value of the bill.

Ans.: BD = 927, BG = 27

TD = BD – BG

TD = 927 – 27 = 900

$F = \frac{BD \times TD}{BG}$

$F = \frac{927 \times 900}{27} = 30,900$

27. If $\cos A = \frac{4}{5}$ find cos 3A.

Ans.: $\cos A = \frac{4}{5}$, $\cos 3A = 4\cos^3 A - 3\cos A$

$\cos 3A = 4 \times \frac{64}{125} - 3 \times \frac{4}{5}$

$\cos 3A = \frac{256}{125} - \frac{12}{5} = \frac{256-300}{925} = -\frac{44}{125}$

28. If $\tan A = \frac{3}{4}$ and $\tan B = \frac{1}{7}$ show that $A + B = \frac{\pi}{4}$.

Ans.: $\tan A = \frac{3}{4}$, $\tan B = \frac{1}{7}$

$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{3}{4} + \frac{1}{7}}{1 - \frac{3}{4} \times \frac{1}{7}} = \frac{\frac{21+4}{28}}{\frac{28-3}{28}}$

$\tan (A + B) = 1 \Rightarrow A + B = \frac{\pi}{4}$.

29. Find the equation of the parabola given that vertex is (0, 0) and focus (3, 0).

Ans.: Focus = (a, 0) = (3, 0) $\Rightarrow$ a = 3

$y^2 = 4ax \Rightarrow y^2 = 12x$

30. If $f(x) = \begin{cases} \frac{x^4 - 256}{x - 4}, & x \neq 4 \\ a, & x = 4 \end{cases}$ is continuous at x = 4, find a.

Ans.: $\lim_{x \to 4} \frac{x^4 - 4^4}{x - 4} = a \Rightarrow 4 \times 4^{4-1} = a \Rightarrow \boxed{256 = a}$

31. If $y = x^{5+\log x}$ find $\frac{dy}{dx}$.

Ans.: $y = x^{5+\log x}$

$\log y = (5 + \log x) \times \log x$

$\frac{1}{y}\frac{dy}{dx} = (5 + \log x) \times \frac{1}{x} + \log x \times \left(0 + \frac{1}{x}\right)$

$\frac{dy}{dx} = x^5 + \log x \left[\frac{5 + \log x}{x} + \frac{\log x}{x}\right]$

32. The displacement 's' of a particle at time 't' is given by $s = 2t^3 - 5t^2 + 4t - 3$ find the velocity at time t = 2 seconds.

Ans.: $S = 2t^3 - 5t^2 + 4t - 3$

$V = \frac{ds}{dt} = 6t^2 - 10t + 4$

At $t = 2$, $\frac{ds}{dt} = 6 \times 4 - 10 \times 2 + 4 = 24 - 20 + 4 = 8$ units.

33. Evaluate : $\int \frac{2x+5}{x^2+5x+3}dx$.

Ans.: $\int \frac{2x+5}{x^2+5x+3}dx \Rightarrow \int \frac{f'(x)}{f(x)}dx = \log[f(x)+C] = \log\left[x^2+5x+3\right]+C$

34. Evaluate : $\int_1^2 \left(2x^2+\frac{1}{x}\right)dx$

Ans.: $\frac{2x^3}{3}+\log x\Big]_1^2 = \left[\frac{16}{3}+\log 2\right]-\left[\frac{2}{3}+\log 1\right]$

$= \frac{14}{3}+\log 2-\log 1 = \frac{14}{3}+\log 2$

PART – C

V. Answer any nine questions : **(9 × 3 = 27)**

35. Solve using Cramer's Rule :

$3x + 2y = 8$

$4x - 3y = 5$

Ans.: $3x + 2y = 8$

$4x - 3y = 5$

$\Delta = \begin{vmatrix} 3 & 2 \\ 4 & -3 \end{vmatrix} = -9-8 = -17$

$\Delta x = \begin{vmatrix} 8 & 2 \\ 5 & -3 \end{vmatrix} = -24-10 = -34$

$\Delta x = \begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} = 15-32 = -17$

$x = \frac{\Delta x}{\Delta} = \frac{-34}{-17} = 2;\ y = \frac{\Delta y}{\Delta} = \frac{-17}{-17} = 1$

36. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

Ans.: $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = C_1^1 \to C_1 - C_2\ ; C_2^1 \to C_2 - C_3$

$$\begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ (a+b)(a-b) & (b+c)(b-c) & c^2 \end{vmatrix} = (a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & C \\ a+b & b+c & c^2 \end{vmatrix}$$

$= (a - b)\ (b - c)\ \ (c - a)$

37. A team of 8 players has to be selected from 14 players. In how many ways the selections can be made if

a) Two particular players are always selected.

b) Two particular players are always excluded.

c) Any 8 players are selected from 14 players.

Ans.: Total = 4 = 14 ; Req = r = 8

a) ${}^{n-2}C_{r-2} = {}^{12}C_6$

b) ${}^{n-2}C_r = {}^{12}C_8$

c) No. of ways = ${}^{14}C_8$

38. A card is drawn from a pack of 52 playing cards. What is the probability that the card is king given that the card is red ?

Ans.: A : card is king

n(A) = 4

B : Card is red

$n(A \cap B) = 2$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)} = \frac{2}{26} = \frac{1}{13}$$

39. Two taps can separately fill a tank in 12 minutes and 15 minutes separately. The tank when full can be emptied by a drain pipe in 20 minutes when the tank was empty, all the three taps were opened simultaneously. In what time will the tank be filled up ?

Ans.: 1 min work of 1st tap = $\frac{1}{12}$

1 min work of 2nd tap = $\frac{1}{15}$

1 min work of drain pipe = $\frac{1}{20}$

When 3 pipes opened,

1 min work together = $\frac{1}{12} + \frac{1}{15} - \frac{1}{20} = \frac{5+4-3}{60} = \frac{6}{60} = \frac{1}{10}$

Time required = 10 minutes

40. A bill for Rs. 2,920 drawn at 6 months was discounted on 10-4-97 fro Rs. 2,916. If the discount rate is 5% p.a. On what date was the bill drawn ?

Ans.: F = Rs. 2,920, DV = Rs. 2,916, r = 0.05

Legally due date = ?

Date of drawing = ?

Discounted date = 10 – 4 – 97

D. V. = F (1 – tr)

2,916 = 2,920 (1 – t × 0.05)

0.05t = 1 – 0.998630

0.05t = 0.00137

t = 10 days

Legally due date = 10 days after 10 – 4 – 97

Legally due date = 20 – 4 – 97

$\therefore$ Date of drawing = Legally due date

- Bill period

- grace period

= 20 – 4 – 97

– 0 – 6 – 0

– 3 – 0 – 0

17 – 10 – 96

41. What is the market value of 12% stock when an investment of Rs. 6,900 produces an income of Rs. 720.

Ans.:

M. V.	**Income**
6,900	720
x	12

M. V. = $\frac{12\times6,900}{720}$ = Rs.115

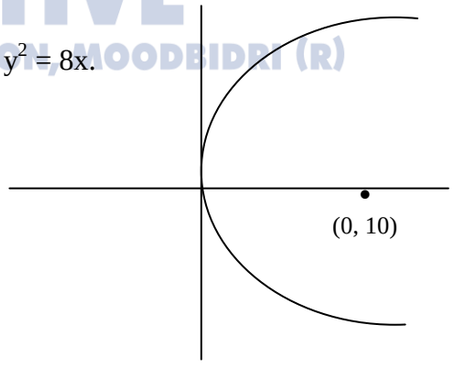
42. Gopal purchased a scooter costing Rs. 32,450. If the rate of sales tax is 9% calculate the total amount payable by him.

Ans.: SP = MP $\left(\frac{100+ST\%}{100}\right)$

SP = 32,450 $\left(\frac{100+9}{100}\right)$

SP = **Rs. 35,370.50**

43. Find, directrix, focus and vertex of the parabola $y^2 = 8x$.



Ans.: y2 = 8x

4a = 8 $\Rightarrow$ a = 2

Focus = (a, 0) = (2, 0)

Equation of directrix, x = –2

Vertex = (0, 10)

44. If x = aθ, y = $\frac{a}{\theta}$ then, prove that $\frac{dy}{dx}+\frac{y}{x}=0$.

Ans.: x = aθ, $\frac{dx}{d\theta}=a$

$y=\frac{a}{\theta}\ \frac{dy}{d\theta}=-\frac{-a}{\theta^2}\quad \frac{dy}{dx}=\frac{\frac{-a}{\theta^2}}{a}=\frac{-1}{\theta^2}$

$$\frac{dy}{dx}+\frac{y}{x}=\frac{-1}{\theta^2}+\frac{\frac{a}{\theta}}{a\theta}=\frac{-1}{\theta^2}+\frac{1}{\theta^2}=0$$

45. A square plates is expanding uniformly, the side is increasing at the rate of 5 cm/sec, what is the rate at which the area is increasing is increasing when the side is 20cm long ?

Ans.: $\frac{dx}{df}=5\,\text{c.m/sec.}, x=20\,\text{c.m.}$

$A = x^2$

$$\frac{dA}{dt}=2x\times\frac{dx}{dt}=2\times 20\times 5=200\,\text{c.m}^2/\text{sec}$$

46. Divide the number 40 into two parts such that their product is maximum.

Ans.: Let the numbers be x and y

$x + y = 40 \Rightarrow y = 40 - x$

P = xy is maximum

$P = x(40 - x^2)$

$P = 40x - x^2$

$$\frac{dp}{dx}=40-2x\Rightarrow\frac{dp}{dx}=0\Rightarrow 40-2x=0\Rightarrow x=20$$

$\frac{d^2p}{dx^2}=-2>0$, P has maximum at x = 20.

$\therefore$ y = 40 – 20 = 20

$\therefore$ x = 20, y = 20.

47. Evaluate : $\int x\cos x\,dx.$

Ans.: u = x ; v = cos x

$x' = 1 \quad \int v = \sin x$

$\int u\ell = u\int\ell - \int u'\int\ell\,dx = x\sin x - \int 1\times\sin x = x\sin x + \cos x + C$

48. Evaluate : $\int_0^1 (6x+1)\sqrt{3x^2+x+5}\,dx.$

$$\int_0^1 (6x+1)\sqrt{3x^2+x+5}\,dx$$

Sub : $3x^2 + x + 5 = t$

(6x + 1) dx = dt

x = 0, t = 5

x = 1, t = 3 + 1 + 5 = 9

$$\int_5^9 t^{\frac{1}{2}}dt=\left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right]_5^9=\frac{2}{3}\times\left[9^{\frac{3}{2}}-5^{\frac{3}{2}}\right]$$

$$= \frac{2}{3}\left[3^3 - \left(\sqrt{5}\right)^3\right] = \frac{2}{3}\left[27 - 5\sqrt{5}\right]$$

PART – D

VI **Answer any five questions :** **(5 × 5 = 25)**

49. Solve the following system of linear equations by matrix method.

$3x - y + 2z = 13$

$2x + y - z = 3$

$x + 3y - 5z = -8$

Ans.: $3x - y + 2z = 13$

$2x + y - z = 3$

$x + 3y - 5z = -8$

$$A = \begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & -1 \\ 1 & 3 & -5 \end{bmatrix} \times = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 13 \\ 3 \\ -8 \end{bmatrix}$$

$|A| = 3(-2) + 1(-9) + 2(5) = -5 \neq 0$, A^{-1} exist.

$$\text{Adj } A = \begin{bmatrix} -2 & 9 & 5 \\ 1 & -17 & -10 \\ -1 & 7 & 5 \end{bmatrix} = \begin{bmatrix} -2 & 1 & -1 \\ 9 & -17 & 7 \\ 5 & -10 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}.\text{adj}\,A = \frac{1}{-5}\begin{bmatrix} -2 & 1 & -1 \\ 9 & -17 & 7 \\ 5 & -10 & 5 \end{bmatrix}$$

$$x = A^{-1}B = \frac{1}{-5}\begin{bmatrix} -2 & 1 & -1 \\ 9 & -17 & 7 \\ 5 & -10 & 5 \end{bmatrix}\begin{bmatrix} 13 \\ 3 \\ 8 \end{bmatrix} = \frac{1}{-5}\begin{bmatrix} -15 \\ 10 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} \Rightarrow x = 3, y = -2, z = 1$$

50. Find the coefficient of x'' in $\left(x + \frac{2}{x^2}\right)^{17}$.

Ans.: $\left(x + \frac{2}{x^2}\right)^{17}$

$a = x$, $b = 2x^{-2}$ $n = 17$

$T_{r+1} = {}^nC_r\ a^{n-r}\ b^r$

$T_{r+1} = {}^{16}C_r\ x^{17-r} \times (2x^{-2})^2$

$T_{r+1} = {}^{17}C_r \times 2^r \times x^{17-r-2r}$

$T_{r+1} = {}^{17}C_r \times 2^r \times x^{17-3r}$

Comparing $x^{17-3r} = x^{11}$

$3r = 6 \Rightarrow r = 3$

Co-efficient = $^{17}C_3 \times 2^3$

51. Resolve $\frac{3x+2}{(x-2)(x+3)^2}$ into partial fractions.

Ans.: $\frac{3x+2}{(x-2)(x+3)^2} = \frac{A}{x-2} + \frac{B}{(x+3)} + \frac{C}{(x+3)^2}$

$3x + 2 = A(x + 3)^2 + B(x - 2)(x + 3) + C(x - 2)$

Put $x = -3$

$-9 + 2 = C(-3 - 2) \Rightarrow -7 = -56 \Rightarrow C = \frac{7}{5}$

Put $x = 2$

$8 = 25A \Rightarrow A = \frac{8}{25}$

Comparing co-efficient of x^2,

$0 = A + B \Rightarrow B = -A = -\frac{8}{25}$

$\therefore \frac{3x+2}{(x-2)(x+3)^2} = \frac{\frac{8}{25}}{x-2} + \frac{\frac{-8}{25}}{(x+3)} + \frac{\frac{7}{5}}{(x+3)^2}$

52. Verify whether the proposition $(p \wedge \sim q) \wedge (\sim p \vee q)$ is a contradiction or not.

Ans.:

p	**q**	**~ q**	**(a) p ∧ ~ q**	**~ q**	**(b) ~ p ∨ q**	**a ∧ b**
T	T	F	F	F	T	F
T	F	T	T	F	F	F
F	T	F	F	T	T	F
F	F	T	F	T	F	F

Given proposition is a contradiction

53. If two men and four women can do a work in 33 days and 3 men and 5 women can do the same work in 24 days. How long shall 5 men and 2 women do the same work ?

Ans.: 2 Men + 4 women = 33 days …….. (1)

66 M + 132 N = 1 day ….⊗

3 Men + 5 women = 24 days

72 M + 120 women = 1 day ….⊗

Comparing, 1M = 2W

$\therefore$ 2M + 2M = 4 men – 33 days

5M + 2W = 5M + 1M = 6 Men – x days

Men	Days
4 ↑	33 ↓
6	x

$$\frac{10}{4}=\frac{x}{33}=\frac{4}{6}=\frac{x}{33}\Rightarrow x=\frac{33\times 4}{6}=22\,\text{days}$$

54. An engineering company has 80% learning effect and spends 800 hours to produce 1 lot of the product. Estimate the labour cost for producing 8 lots of the product at the rate of Rs. 20 per hour.

Ans.:

Units produced	Total output in units	Cumulative arrange time per unit	Total hours
1	1	800	800
1	2	80% 800 = 640	1280
2	4	80% 640 = 512	2048
4	8	80% 512 = 409.6	3276.8

Total hours = 32,768

Total cost = 3276 × 20 = Rs. 65536

55. Solve the following LPP graphically :

Maximize : Z = 10500 x + 9000y,

Subject to the constraints : 2x + y ≤ 80

x + y ≤ 50 and x ≥ 0, y ≥ 0.

Ans.: Z = 10500x + 9000 y

2x + y = 80

x	0	40
y	80	0

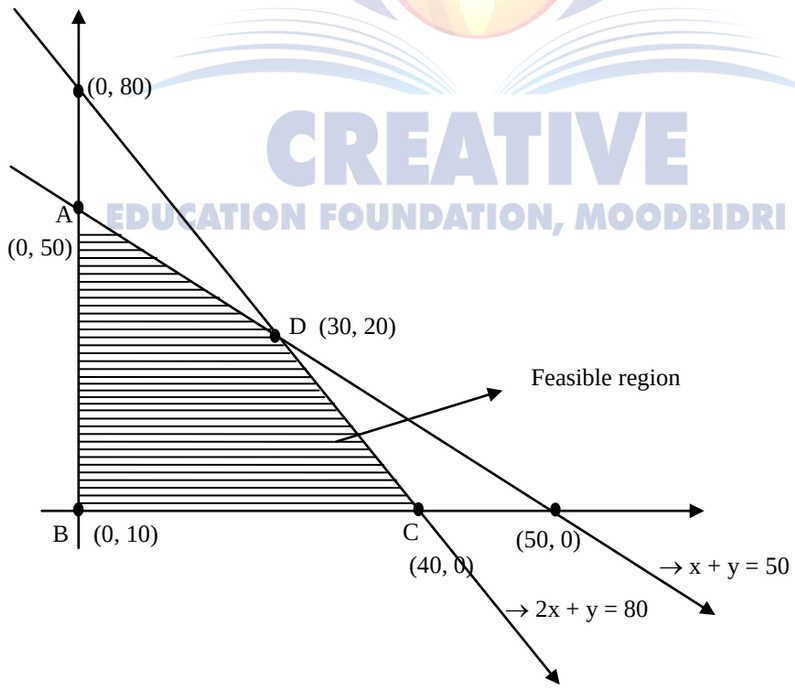
x + y = 50

x	0	50
y	50	0

x ≥ 0, y ≥ 0

2x + y = 80

x + y = 50



Points	z = 10500 x + 9000 y

(0, 50)	4,50,000
(0, 0)	0
(40, 0)	4,20,000
(30, 20)	3,50,000 + 1,80,000 = 4,95,000

Z_{max} = 4,95,000 occurs at (30, 20)

56. Prove that $\frac{\sin 5A+\sin 4A+\sin 2A+\sin A}{\cos 5A+\cos 4A+\cos 2A+\cos A}=\tan 3A.$

Ans.: $\frac{\sin 5A+\sin 4A+\sin 2A+\sin A}{\cos 5A+\cos 4A+\cos 2A+\cos A}$

$=\frac{(\sin 5A+\sin A)+(\sin 4A+\sin 2A)}{(\cos 5A+\cos A)+(\cos 4A+\cos 2A)}$

$=\frac{2\sin(3A).\cos(2A)+2\sin 3A\cos A}{2\cos 3A\cos 2A+2\cos 3A\cos A}$

$=\frac{2\sin 3A(\cos 2A+\cos A)}{2\cos 3A(\cos 2A+\cos A)}=\tan 3A$

57. If $y = \log\left(x+\sqrt{x^2+1}\right)$, show that $(x^2 + 1)\, y_2 + xy_1 = 0$.

Ans.: $y=\log\left(x+\sqrt{x^2+1}\right)$

$y_1=\frac{1}{x+\sqrt{x^2+1}}\times\left(1+\frac{1}{2\sqrt{x^2+1}}\times 2x\right)$

$y_1=\frac{1}{x+\sqrt{x^2+1}}\times\left(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\right)\Rightarrow y_1=\frac{1}{\sqrt{x^2+1}}$

$y_1=\sqrt{x^2+1}=1\Rightarrow y_1.\frac{1}{2\sqrt{x^2+1}}\times 2x+\sqrt{x^2+1}.y_2=0$

$\frac{xy_1}{\sqrt{x^2+1}}+\sqrt{x^2+1}\,y_2=0\Rightarrow xy_1+(x_2+1)y_2=0$

58. Find the area bounded by the parabola $y^2 = 4x$ and the line $x - y = 0$.

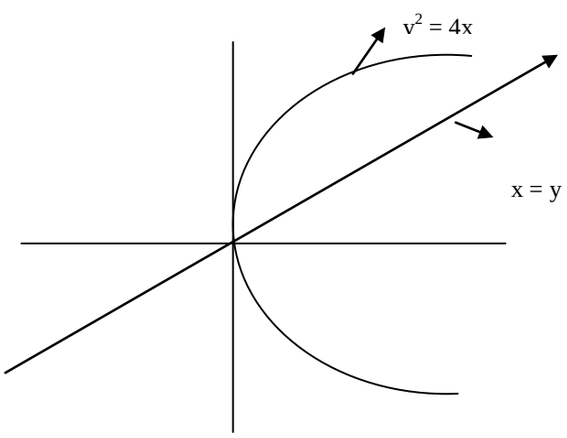
Ans.: $y^2 = 4x$ & $x - y = 0$

$y^2 = 4x$ & $x = y$

$x^2 - 4x = 0$

$x(x - 4) = 0$

$x = 0, x = 4$



$A=\int_0^4 y_1\,dx-\int_0^4 y_2\,dx\Rightarrow A\int_0^4 2.\sqrt{x}\,dx-\int_0^4 x\,dx$

$A=2.\frac{2}{3}\,x^{\frac{3}{2}}-\frac{x^2}{2}\Big]_0^4=\left(2\times\frac{2}{3}\times 4^{\frac{3}{2}}-\frac{16}{2}\right)-[0]$

$$= \frac{4}{3} \times 8 - 8 = \frac{32}{3} - 8 = \frac{32-24}{3} = \frac{8}{3} \text{ sq.units}$$

PART – E

VII. Answer the following :

59. a) Show that the points (0, 0), (1,1), (5, –5) and (6, –4) are concyclic. **(6)**

OR

b) If the angle 'θ' is measured in radians. Prove that $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$.

(6)

Ans.: Equation circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

$(0, 0) \Rightarrow C = 0$

$(1, 1) \Rightarrow 1 + 1 + 2y + 2f = 0 \Rightarrow 2y + 2f = -2$

$\Rightarrow g + f = -1$ …….. (2)

$(5, -5) \Rightarrow 25 + 25 + 10g - 10f = 0$

$10g - 10f = -50$

$g - f = -5$ …….. (3)

Solving : (2) and (3)

$g + f = -1$

$g - f = -5$

$\underline{2g = -6}$

$\mathbf{\underline{g = 3}}$

$-3 + f = -1$

$\mathbf{\underline{f = 2}}$

Sub in (1) $x^2 + y^2 - 6x + 4y = 0$

Consider (6, –4) & $x^2 + y^2 - 6x + 4y$

$36 + 16 - 36 - 16 = 0$

$\therefore$ Points are con – cyclic

b) Consider a unit circle.

Draw AB $\perp^{lr}$ OA & CD $\perp^{lr}$ OA

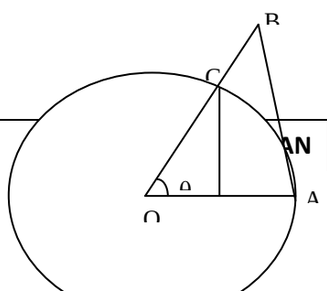
Here OA = OC = 1 units.

Case 1 : Let 'O' be a positive angle.

Here Area of Δ^{le} AOC < Area of section AOC < Area of OAB

$$\frac{1}{2} \times 1 \times CD < \frac{1}{2} r^2\theta < \frac{1}{2} \times 1 \times AB$$

$\Rightarrow CD < \theta < AB$ ……. (1)



From Δ^{le} ODC, CD = $\sin\theta$

From Δ^{le} OAB, AB = $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Sub in (1)

$\sin\theta < \theta < \frac{\sin\theta}{\cos\theta}$

$1 < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta}$

$\cos\theta < \frac{\sin\theta}{\theta} < 1$

$\lim_{\theta\to 0}\cos\theta \le \lim_{\theta\to 0}\frac{\sin\theta}{\theta} \le \lim_{\theta\to 0} 1\ ;\ 1 \le \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$

Case : 2 'θ' is "– 've : Let θ be '–' ve then '–θ' is '+' ve

Consider $\frac{-\sin\theta}{-\theta}$

Taking limits, $\lim_{\theta\to 0}\frac{-\sin\theta}{-\theta} = \lim_{\theta\to 0}\frac{\sin(-\theta)}{-\theta} = 1 (\text{bycase 1})$

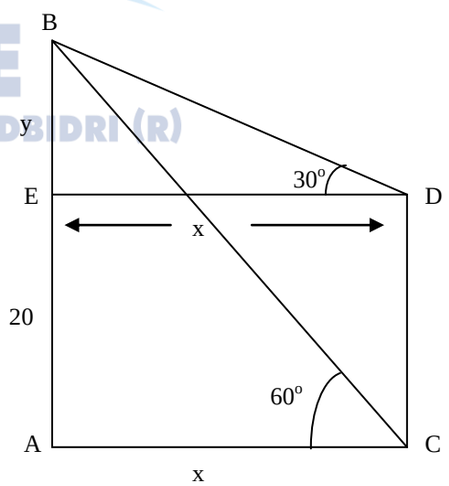
$\therefore \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$

60. a) The angle of elevation of the top of a tower from the base and the top of a building are 600 and 300. The building is 20m high. Find the height of the tower. **(4)**

OR

b) Find the value of $(1.01)^5$ using Binomial theorem, upto 4 decimal places. **(4)**

Ans.: **a)** From Δ^{th} BED



$\tan 30^o = \frac{y}{x}$

$\frac{1}{\sqrt{3}} = \frac{y}{x} \Rightarrow x = y\sqrt{3}$

From Δ^{th} BAC, $\tan 60^o = \frac{20+y}{x}$

$\sqrt{3}\times x = 20 + y \Rightarrow \sqrt{3}\times y\sqrt{3} = 20 + y$

$\Rightarrow 3y = 20 + y \Rightarrow y = 10m \Rightarrow$ height = 20 + y = 30m

b) $(1.01)^5 = (1 + 0.01)^5$

$= {}^5C_0 + {}^5C_1 \times 0.01 + {}^5C_2 \times (0.01)^2 + {}^5C_3 \times (0.01)^3 + {}^5C_4 \times (0.01)^4 + {}^5C_5 \times (0.01)^5$

$= 1 + 0.05 + 10 \times (0.01)^2 + 10 \times (0.01)^3 + 5 \times (0.01)^4 + 1 \times (0.01)^5$

$= 1.05101.$

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