## CREATİVE LEARNING CLASSES KARKALA

## Sapthagiri Campus, Kanangi Road, Hirgana - 576117

## 2023-24 II PUC ANNUAL EXAMINATION

BASIC MATHEMATICS

PART-A

I. Answer all the multiple choice questions :

1. If $\mathbf{A}=\left[\begin{array}{ccc}1 & 2 & 4 \\ -1 & 3 & -2\end{array}\right]$ and $\mathbf{B}=\left[\begin{array}{ccc}3 & -4 & -1 \\ 1 & 5 & -2\end{array}\right]$ the $(\mathbf{A}+\mathbf{B})$ is
а) $\left[\begin{array}{ccc}4 & 2 & -3 \\ 0 & 8 & 4\end{array}\right]$
b) $\left[\begin{array}{lll}4 & -2 & -3 \\ 0 & -8 & -4\end{array}\right]$
c) $\left[\begin{array}{ccc}-4 & 2 & 3 \\ 0 & -8 & 4\end{array}\right]$
d) $\left[\begin{array}{lll}4 & -2 & -3 \\ 0 & -8 & -4\end{array}\right]$

Solution : Option : (b)
$A+B=\left[\begin{array}{ccc}1 & 2 & 4 \\ -1 & 3 & -2\end{array}\right]+\left[\begin{array}{ccc}3 & -4 & -1 \\ 1 & 5 & -2\end{array}\right]=\left[\begin{array}{lll}4 & -2 & -3 \\ 0 & -8 & -4\end{array}\right]$
2. If ${ }^{\mathrm{n}} \mathrm{C}_{\mathbf{1 0}}={ }^{\mathrm{n}} \mathrm{C}_{\mathbf{1 5}}$ then n is
a) 25
b) 29
c) $\mathbf{2 4}$
d) 23

Solution : Option (a)
${ }^{\mathrm{n}} \mathrm{C}_{10}={ }^{\mathrm{n}} \mathrm{C}_{15} \Rightarrow \mathrm{n}=10+15=25$
3. The probability of getting a black card from a pack of $\mathbf{5 2}$ cards is
a) $\frac{3}{4}$
b) $\frac{1}{52}$
c) $\frac{1}{4}$
d) $\frac{1}{2}$

Solution : Option (d)
$\mathrm{P}(\mathrm{A})=\frac{26}{52}=\frac{1}{2}$
4. The value of $4 \cos ^{3} 10^{0}-3 \cos 10^{\circ}$ is
a) $\frac{\sqrt{3}}{2}$
b) $\frac{2}{\sqrt{3}}$
c) $\frac{1}{\sqrt{3}}$
d) $\frac{1}{2}$

Solution : Option (a)
$4 \cos ^{3} 10^{\circ}-3 \cos 10^{\circ}=\cos \left(3 \times 10^{\circ}\right)=\cos 30^{\circ}=\frac{\sqrt{3}}{2}$.
5. The value of $\int 4 \sec ^{2} x d x$ is
a) $4 \sec x+c$
b) $4 \sin x+c$
c) $4 \boldsymbol{\operatorname { t a n }} x+c$
d) $4 \cot x+c$

Solution : Option (c)
$\int 4 \sec ^{2} x d x=4 \tan x+c$
II. Match the following
6. i) The value of $\left|\begin{array}{ll}3200 & 3201 \\ 3202 & 3203\end{array}\right|$ is $\quad$ a) 27
ii) If ${ }^{5} \mathrm{P}_{\mathrm{r}}=60$, then r is
b) 12
iii) If 5:20 $=3: x$ then the value of $x$ is
c) $\frac{y}{x}$
iv) The value of $\lim _{x \rightarrow 3}\left(\frac{x^{3}-27}{x-3}\right)$ is
d) $\frac{x}{y}$
v) If $x^{2}-y^{2}=a^{2}$ then $\frac{d y}{d x}$ is
e) -2
f) 3

## Solution :

i) $\left|\begin{array}{ll}3200 & 3201 \\ 3202 & 3203\end{array}\right|=\left|\begin{array}{ll}3200 & 1 \\ 3202 & 1\end{array}\right|=3200-3202=-2$
ii) ${ }^{5} \mathrm{P}_{\mathrm{r}}=60 \Rightarrow$ for $\mathrm{r}=3,5 \times 4 \times 3=60 . \quad \therefore \mathrm{r}=3$.
iii) $5: 20=3: x$

$$
\frac{5}{20}=\frac{3}{x} \Rightarrow x=12
$$

iv) $\lim _{x \rightarrow 3}\left(\frac{x^{3}-27}{x-3}\right)=\operatorname{lt}_{x \rightarrow 3}\left(\frac{x^{3}-3^{3}}{x-3}\right)=3 \times 3^{2}=27$
v) $x^{2}-y^{2}=a^{2}$ then $\frac{d y}{d x}$ is
$2 x-2 y \cdot \frac{d y}{d x}=0 \Rightarrow \frac{2 x}{2 y}=\frac{d y}{d x} \Rightarrow \frac{d y}{d x}=\frac{x}{y}$
III. For question numbers 7 to 11 choose the appropriate answer from the brackets given below :
$\left(56,9, \frac{-3}{4}, 1,2,4\right)$
7. If $\left[\begin{array}{lll}2 & x & 2\end{array}\right]\left[\begin{array}{l}1 \\ 4 \\ 2\end{array}\right]=[3]$ then the value of $x$ is
8. The number of triangles that can be formed from the 8 non collinear points is $\qquad$
9. The third proportional of 4 and 6 is $\qquad$
10. The value of $\lim _{x \rightarrow 0}\left(\frac{\sin 4 x}{\sin 2 x}\right)$ is $\qquad$
11. The value of $\int_{0}^{\pi / 2} \sin 2 x d x$ is $\qquad$

## Solution :

7. $\left[\begin{array}{lll}2 & \mathrm{x} & 2\end{array}\right]\left[\begin{array}{l}1 \\ 4 \\ 2\end{array}\right]=[3]$

$$
\begin{aligned}
& 2+4 x+4=3 \\
& 4 x=-3 \\
& x=\frac{-3}{4}
\end{aligned}
$$

8. Number of triangles $={ }^{8} \mathrm{C}_{3}=\frac{8 \times 7 \times 6}{3 \times 2 \times 1}=56$
9. $4: 6:: 6: x$

$$
4 x=36 \Rightarrow x=9 .
$$

10. $\operatorname{lt}_{x \rightarrow 0} \frac{\sin 4 x}{\sin 2 x}=\operatorname{lt}_{x \rightarrow 0} \frac{4 x}{2 x}=2$
11. $\left.\int_{0}^{\pi / 2} \sin 2 x=-\frac{\cos 2 x}{2}\right]_{0}^{\frac{\pi}{2}}=-\frac{1}{2}[\cos x-\cos 0]=\frac{-1}{2}[-1-1]=1$
IV. Answer the following questions :
12. Negate : $\sim \mathrm{p} \rightarrow \mathrm{q}$

Solution : $\sim(\sim p \rightarrow q)=\sim p \wedge \sim q$
$(\because \sim(\mathrm{a} \rightarrow \mathrm{b})=\mathrm{a} \wedge \sim \mathrm{b}))$
13. If $\mathrm{a}: \mathrm{b}=2: 3, \mathrm{~b}: \mathrm{c}=5: 7$ and $\mathrm{c}: \mathrm{d}=3:$ then find $\mathrm{a}: \mathrm{d}$.

Solution : $\frac{\mathrm{a}}{\mathrm{d}}=\frac{\mathrm{a}}{\mathrm{b}} \times \frac{\mathrm{b}}{\mathrm{c}} \times \frac{\mathrm{c}}{\mathrm{d}} \Rightarrow \frac{\mathrm{a}}{\mathrm{d}}=\frac{2}{3} \times \frac{5}{7} \times \frac{3}{1}=\frac{10}{7}$
$\mathrm{a}: \mathrm{d}=10: 7$
14. If $\tan \mathrm{A}=\frac{1}{\sqrt{3}}$ the find $\tan 2 \mathrm{~A}$.

Solution : $\tan \mathrm{A}=\mathrm{y} \sqrt{3} \Rightarrow \mathrm{~A}=300$
$\tan 2 \mathrm{~A}=\tan 60^{\circ}=\sqrt{3}$
15. Differentiate $3 x^{2}+4 y^{2}=10$ w.r.t.x.

Solution: $3 x^{2}+4 y^{2}=10$
Diff w.r.t.' $x$ '
$6 x+8 y \cdot \frac{d y}{d x}=10 \Rightarrow \Rightarrow 8 y \frac{d y}{d x}=-6 x \Rightarrow \frac{d y}{d x}=\frac{-6 x}{8 y}=\frac{-3 x}{4 y}$
16. Evaluate $\int\left(x^{2}-\frac{6}{x}+5 e^{x}\right) d x$

Solution : $\int\left(x^{2}-\frac{6}{x}+5 e^{x}\right) d x=\frac{x^{3}}{3}-6 \log x+5 e^{x}+c$
PART - B
V)Answer any SIX questions
17. In how many ways can the letters of the word "HOPPER" be arranged?

Solution : HOPPER
$\mathrm{n}=6 \mathrm{p}=2$
No. of ways $=\frac{6!}{2!}=\frac{720}{2}=360$
18. Find the number of parallelograms that can be formed from the set of $\mathbf{6}$ parallel lines intersecting another set of 4 parallel lines.

Solution : $m=6 n=4$
Name of parallelograms $={ }^{\mathrm{m}} \mathrm{C}_{2} \times{ }^{\mathrm{n}} \mathrm{C}_{2}={ }^{6} \mathrm{C}_{2} \times{ }^{4} \mathrm{C}_{2}=\frac{6 \times 5}{2 \times 1} \times \frac{4 \times 3}{2 \times 1}=15 \times 6=90$
19. Two coins are tossed simultaneously. What is the probability of getting
a) Atleast one tail
b) Atmost one tail

Solution : S = $\{\mathrm{HH}, \mathrm{HT}, \mathrm{TH}, \mathrm{TT}\}$
a) $p($ atleast one tail $)=\frac{3}{4}$
b) $\mathrm{P}($ atymost one tail $)=\frac{3}{4}$
20. Divide Rs. 6,000 in the ratio $3: 4: 5$.

Solution : given ratio 3:4:5
Let the parts are $3 \mathrm{x}, 4 \mathrm{x}, 5 \mathrm{x}$.
Given, $3 \mathrm{x}+4 \mathrm{x}+5 \mathrm{x}=6,000$
$12 \mathrm{x}=6000 \Rightarrow \mathrm{x}=500$
$1^{\text {st }}$ part $=3 \times 500=1500$
$2^{\text {nd }}$ part $=4 \times 500=2000$
$3^{\text {rd }}$ part $=5 \times 500=2500$
21. 500 workers can finish a work in 8 days. How many workers will finish the same work in 5 days?
Solution :

$\left.$| Workers | Days |  |
| :--- | :--- | :---: |
| 500 |  |  |
| X |  |  |\(\uparrow \quad \begin{gathered}8 <br>

5\end{gathered} \right\rvert\,\)
Workers and days are in inverse proportion
$\therefore \frac{500}{\mathrm{x}}=\frac{5}{8}$
$5 \mathrm{x}=500 \times 8$
$x=800 \quad \therefore$ No. of workers $=800$
22. For Rs. $\mathbf{5 1 2 . 5 0}$ due $\mathbf{6}$ months at $\mathbf{1 5 \%}$ p.a. Find the true present value and discounted value of the bill.

Solution :
$\mathrm{F}=$ Rs. 512.50
$\mathrm{t}=6$ months $=0.5$ years
$r=0.15 \%$
Present value $\mathrm{P}=\frac{\mathrm{F}}{1+\mathrm{tr}}=\frac{512.50}{1+0.075}=$ Rs. 476.74
$\mathrm{DV}=\mathrm{F}(1-\mathrm{tr})$
DV $=512.50(1-0.075)=$ Rs. 474.06
23. Find the equation of the parabola given that its focus is $(-4,0)$ and directrix is $x=4$.

Solution : Focus $=(-4,0) \Rightarrow \mathrm{a}=4$
Equation is $y^{2}=-4 a x \Rightarrow y^{2}=-16 x$
24. Find the axis and length of the latus rectum of the parabola $x^{2}=16 y$.

Solution : $x^{2}=16 y$
$4 \mathrm{a}=16 ; \mathrm{a}=4$
Axis $=y-$ axis
$\operatorname{LLR}=4 \mathrm{a}=4 \times 4=16$ units
25. $\int \frac{4 \mathrm{x}+3}{2 \mathrm{x}^{2}+3 \mathrm{x}+5} \mathrm{dx}$

Solution : $\int \frac{4 x+3}{2 x^{2}+3 x+5} d x$
$f(x)=2 x^{2}+3 x+5$
$\mathrm{f}^{\prime}(\mathrm{x})=4 \mathrm{x}+3$
$\int \frac{f^{\prime}(x)}{f(x)} d x=\log (f(x))+c$
$=\log \left(2 x^{2}+2 x+5\right)+c$
26. Evaluate $\int_{0}^{3}\left(\frac{x+3}{x+2}\right) d x$.

Solution : $\int_{0}^{3} \frac{x+3}{x+2} d x=\int_{0}^{3} \frac{x+2+1}{x+2} d x \Rightarrow \int_{0}^{3}\left(1+\frac{1}{x+2}\right) d x$
$=\mathrm{x}+\log (\mathrm{x}+2)]^{3}{ }_{0}=3+\log 5-(0+\log 2)$
$=3+\log 5-\log 2$
27. Find the area enclosed by the curve $y=x^{2} . x-$ axis and the ordinates $x=0$ and $x=1$.

Solution :
$y=x^{2}, x=0, x=1$

$$
\begin{aligned}
& A=\int_{0}^{1} y d x \Rightarrow A=\int_{0}^{1} x^{2} d x \\
& \left.\Rightarrow \frac{x^{3}}{3}\right]_{0}^{1} \Rightarrow A=\frac{1}{3}
\end{aligned}
$$



## PART - C

VI. Answer any FIVE of the following questions :
28. Solve :
$3 x+2 y=8$ and $4 x-3 y=5$ by Cramer's rule.
Solution :
$3 x+2 y=8$
$4 x-3 y=5$
$\mathrm{A}=\left|\begin{array}{cc}3 & 2 \\ 4 & -3\end{array}\right|=-17$
$\Delta x=\left|\begin{array}{cc}8 & 2 \\ 5 & -3\end{array}\right|=-34$
$\Delta y=\left|\begin{array}{ll}3 & 8 \\ 4 & 5\end{array}\right|=-17$
$\mathrm{x}=\frac{\Delta \mathrm{x}}{\Delta}=\frac{-34}{-17}=2$
$y=\frac{\Delta y}{\Delta}=\frac{-17}{-17}=1$
29. The difference between BD and TD on a certain sum of money due in $\mathbf{6}$ months is Rs.
27. Find the amount of the bill if the rate of interest is $6 \%$ p.a.

Solution : BD - TD = 27
$\mathrm{BG}=27$
$\mathrm{t}=6$ months $=0.05$ years
$\mathrm{r}=0.06$
$\mathrm{BG}=\mathrm{TD} . \operatorname{tr}$
$27=$ TD $.0 .5 \times 0.06$
$\mathrm{TD}=900$
$B D-900=27$
BD $=927$
$\mathrm{F}=\frac{\mathrm{BD} \times \mathrm{TD}}{\mathrm{BG}}=\frac{927 \times 900}{27} \Rightarrow \mathrm{~F}=\mathrm{Rs} .30,900$
30. A person invests Rs. $\mathbf{1 5 , 0 0 0}$ partly in $\mathbf{3 \%}$ stock at 75 and partly in $\mathbf{6 \%}$ stock at $\mathbf{1 2 5}$. If the income from both is Rs. 675. Find his investment in 2 types of stocks.
Solution : Money invested in $3 \%$ stock $=\mathrm{x}$
Money invested in $6 \%$ stock $=15,000-\mathrm{x}$
Income $I_{1}=\frac{x \times 3}{75}=0.04 x$
Income $\mathrm{I}_{2}=\frac{(15,000-\mathrm{x}) \times 6}{125}$
$\mathrm{I}_{2}=720-0.048 \mathrm{x}$
$\mathrm{I}_{1}+\mathrm{I}_{2}=675$
$0.04 \mathrm{x}+720-0.048 \mathrm{x}=675$
$0.008 \mathrm{x}=45$
$\mathrm{x}=$ Rs. 5625
$\therefore$ Money invested in $3 \%$ stock $=$ Rs. 5625
Money invested in 6\% stock $=1500-5625=$ Rs. 9375
31. The price of a T.V. set inclusive of sales tax of $\mathbf{9 \%}$ is Rs. $\mathbf{1 3}, \mathbf{4 0 7}$. Find its marked price. If the sales tax is increased to $\mathbf{1 3 \%}$, how much more does the customer pay for the T.V.?

Solution : SP of T.V. = Rs. 13,407
$\mathrm{SP}=\mathrm{MP}+\mathrm{ST} \% \mathrm{MP}$
$13,407=x+\frac{9}{100} x$
$13,407=1.09 \mathrm{x}$
$\mathrm{MP}=$ Rs. 12,300
If sales tax is $13 \%$
Then SP = MP + ST \% MP
$\mathrm{SP}=12,300+\frac{13}{100}(12,300)$
SP = Rs. 13,899/-
32. Find $\frac{d y}{d x}$, given that $x=a \cos ^{4} \theta, y=a \sin ^{4} \theta$.

Solution : $a \cos ^{4} \theta \quad y=a \sin ^{4} \theta$

$$
\begin{aligned}
& \frac{d x}{d \theta}=-4 a \cos ^{3} \theta \cdot \sin \theta \quad \frac{d y}{d \theta}=4 a \sin ^{3} \theta \cos \theta \\
& \frac{d y}{d \theta}=\frac{4 a \sin ^{3} \theta \cos \theta}{-4 a \cos ^{3} \theta \sin \theta}=\frac{-\sin ^{2} \theta}{\cos ^{2} \theta}=-\tan ^{2} \theta
\end{aligned}
$$

33. A ladder of $\mathbf{1 5}$ feet long leans against a smooth vertical wall. If the top slides downwards at the rate of $2 \mathrm{ft} / \mathrm{sec}$. Find how fast the lower end is moving when the lower end is $\mathbf{1 2 f e t}$ away from the wall.

Solution : $\frac{\mathrm{dy}}{\mathrm{dt}}=-2 \mathrm{ft} / \mathrm{sec} \frac{\mathrm{dx}}{\mathrm{dt}}=? \quad \mathrm{x}=12$
$x^{2}+y^{2}=15^{2}$
$y^{2}=15^{2}-12^{2}$
$y=9$
$x^{2}+y^{2}=15$

$2 \mathrm{x} \frac{\mathrm{dx}}{\mathrm{dt}}+2 \mathrm{y} \cdot \frac{\mathrm{dy}}{\mathrm{dt}}=0 \Rightarrow 2 \times 12 \times \frac{\mathrm{dx}}{\mathrm{dt}}+2 \times 9 \times(-2)=0$
$\frac{\mathrm{dx}}{\mathrm{dt}}=\frac{+36}{24}=\frac{3}{2} \mathrm{ft} / \mathrm{sec}$
34. Evaluate $\int \frac{x+2}{(2 x-1)(x-3)} d x$.

Solution : $\int \frac{x+2}{(2 x-1)(x-3)} d x=\int \frac{A}{(2 x-1)}+\frac{B}{(x-3)} d x$
$x+2=A(x-3)+B(2 x-1)$
$\mathrm{x}=3, \mathrm{~B}=1$
$\mathrm{x}=\frac{1}{2}, \mathrm{~A}=-1$
$\therefore \int \frac{\mathrm{x}+2}{(2 \mathrm{x}-1)(\mathrm{x}-2)}=\int \frac{\mathrm{x}+2}{(25-1)(\mathrm{x}-2)}=\int \frac{-1}{2 \mathrm{x}-1}+\frac{1}{\mathrm{x}-3} \mathrm{dx}$
$=-\frac{\log (2 x-1)}{2}+\log (x-3)+c$
PART - D
VII. Answer any five questions :
35. Solve the linear equations by matrix method.
$x+y-z=1$
$3 x+y-2 z=3$
$x-y-z=-1$

## Solution :

$x+y-z=1$
$3 x+y-2 z=3$
$\mathrm{x}-\mathrm{y}-\mathrm{z}=-1$
$A x=B \Rightarrow X=A^{-1} B$
Where $A=\left[\begin{array}{ccc}1 & 1 & -1 \\ 3 & 1 & -2 \\ 1 & -1 & -1\end{array}\right], x=\left[\begin{array}{l}x \\ y \\ z\end{array}\right], B=\left[\begin{array}{c}1 \\ 3 \\ -1\end{array}\right]$
$|\mathrm{A}|=\left|\begin{array}{ccc}1 & 1 & -1 \\ 3 & 1 & -2 \\ 1 & -1 & -1\end{array}\right|=2 \neq 0$
$\Rightarrow \mathrm{C}_{11}=-3, \mathrm{C}_{12}=1, \mathrm{C}_{13}=4, \mathrm{C}_{21}=2, \mathrm{C}_{22}=0, \mathrm{C}_{23}=2, \mathrm{C}_{31}=1, \mathrm{C}_{32}=1, \mathrm{C}_{33}=2$
$\mathrm{C}=\left[\begin{array}{ccc}-3 & 1 & -4 \\ 2 & 0 & 2 \\ -1 & -1 & -2\end{array}\right]$
$\operatorname{Adj}(A)=\left[\begin{array}{ccc}-3 & 2 & -1 \\ 1 & 0 & -1 \\ -4 & 2 & -2\end{array}\right]$
$\mathrm{x}=\frac{1}{2}\left[\begin{array}{ccc}-3 & 2 & -1 \\ 1 & 0 & -1 \\ -4 & 2 & -2\end{array}\right]\left[\begin{array}{c}1 \\ 3 \\ -1\end{array}\right]$
$\mathrm{x}=2, \mathrm{y}=1, \mathrm{z}=2$.
36. Find the coefficient $x^{8}$ in $\left(3 x^{2}-\frac{1}{2 x}\right)^{10}$.

Solution : Given $\left(3 x^{2}-\frac{1}{24}\right)^{10}$
$\mathrm{T}_{\mathrm{r}+1}={ }^{\mathrm{n}} \mathrm{C}_{\mathrm{r}} \mathrm{x}^{4-\mathrm{r}} \mathrm{a}^{\mathrm{r}}$
$\mathrm{T}_{\mathrm{r}+1}={ }^{10} \mathrm{C}_{\mathrm{r}}\left(3 \mathrm{x}^{2}\right)^{10-\mathrm{r}}\left(\frac{1}{2 \mathrm{x}}\right)^{\mathrm{r}}$
$\mathrm{T}_{\mathrm{r}+1}={ }^{10} \mathrm{C}_{\mathrm{r}}(3)^{10-\mathrm{r}}\left(\frac{1}{2}\right)^{\mathrm{r}}(\mathrm{x})^{2(10-\mathrm{r})-\mathrm{r}}$
Take $2(10-r)-r=8$
$20-3 \mathrm{r}=8 \Rightarrow 3 \mathrm{r}=12 \Rightarrow \mathrm{r}=4$
Co-eff of $\mathrm{x}^{8}$ is $={ }^{10} \mathrm{C}_{4}(3)^{10-4}\left(\frac{1}{2}\right)^{4}={ }^{10} \mathrm{C}_{4}(3)^{6}\left(\frac{1}{2}\right)^{4}$
37. Resolve $\frac{2 x^{2}+10 x-3}{(x+1)(x-3)(x+3)}$ into partial fractions.

Solution : Given $\frac{2 x^{2}+10 x-3}{(x+1)(x-3)(x+3)}$
$\frac{2 x^{2}+10 x-3}{(x+1)(x-3)(x+3)}=\frac{A}{(x+1)}+\frac{B}{(x-3)}+\frac{C}{(x+3)}$
$2 x^{2}+10 x-3=A(x-3)(x+3)+B(x+1)(x+3)+C(x+1)$
Put $\mathrm{x}=3,2(3)^{2}+10(3)-3=\mathrm{B}(4)(6)$
$18+30-3=24 \mathrm{~B}$
$\frac{45}{24}=\mathrm{B}$
Put $\mathrm{x}=-3,18-30-3=\mathrm{C}(-2)(-6)$
$\frac{15}{12}=\mathrm{C} \Rightarrow \mathrm{C}=\frac{-15}{12}$
Put $x=-1,2-10-3=A(-8)$
$\frac{-11}{-8}=\mathrm{A} \Rightarrow \mathrm{A}=\frac{11}{8}$
$\frac{2 \times 2+10 x-3}{(x+1)(x-3)(x+3)}=\frac{\frac{11}{8}}{(x+1)}+\frac{\frac{45}{24}}{(x-3)}+\frac{\frac{-15}{12}}{x+3}$
38. Show that $\sim(\mathbf{p} \vee \mathbf{q}) \rightarrow(\sim \mathbf{p} \wedge-\mathbf{q})$ is a Tautology.

Solution :

|  |  |  | $(\mathbf{a})$ |  |  | $(\mathbf{b})$ | $\mathbf{a \rightarrow b}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{p}$ | $\mathbf{q}$ | $\mathbf{p} \vee \mathbf{q}$ | $\sim(\mathbf{p} \vee \mathbf{q})$ | $\sim \mathbf{p}$ | $\sim \mathbf{q}$ | $\sim \mathbf{p} \cap \sim \mathbf{q}$ | $\mathbf{T}$ |
| T | T | T | F | F | F | F | T |
| T | F | T | F | F | T | F | T |
| F | T | T | F | T | F | F | T |
| F | F | F | T | T | T | T | T |

$\therefore$ Given proposition is a tautology.
39. ABC company required $\mathbf{1 0 0 0}$ hours to produce $\mathbf{1}^{\text {st }} \mathbf{3 0}$ engines. If the learning effect is $\mathbf{9 0 \%}$. Find the total labour cost at Rs. 20/ hour to produce a total of $\mathbf{1 2 0}$ engines.

Solution : 1 lot $=30$ engine
120 engine $=4$ logs

| Unit produced | Total output time <br> per unit | Cumulative average <br> time per unit | Total labours |
| :---: | :---: | :--- | :---: |
| 1 | 1 | 1000 | 1000 |
| 1 | 2 | $90 \%$ of $1000=900$ | 1800 |
| 2 | 4 | $90 \%$ of $900=810$ | 3240 |

Total hours $=3240$
Total labour cost $=20 \times 3240=$ Rs. $64,800 /-$
40. Solve the following LPP graphically

Mmaximize : $\mathbf{Z}=\mathbf{5 x}+\mathbf{3 y}$
Subject to the constraints :
$3 x+5 y \leq 15$,
$5 \mathrm{x}+2 \mathrm{y} \leq 10$,
$\mathbf{x} \geq 0$,
$\mathrm{y} \geq 0$.
Solution : $\max Z=5 x+3 y$
$3 x+5 y=15$
Put

| $x$ | 0 | 5 |
| :--- | :--- | :--- |
| $y$ | 3 | 0 |

$5 x+2 y=10$

| $x$ | 0 | 2 |
| :--- | :--- | :--- |
| $y$ | 5 | 0 |

We set $3 x+5 y=15$
$5 x+2 y=10$

| Corner points | Value of $\mathbf{Z}$ |
| :--- | :--- |
| $(0,3)$ | 9 |
| $\left(\frac{20}{19}, \frac{45}{19}\right)$ | 12.36 - maximum |
| $(2,0)$ | 10 |

Hence $Z=\frac{235}{19}$ is maximum at $\left(\frac{20}{19}, \frac{45}{19}\right)$
41. Prove that : $\frac{\sin 6 \mathrm{~A}+\sin 2 \mathrm{~A}+2 \sin 4 \mathrm{~A}}{\sin 7 \mathrm{~A}+\sin 3 \mathrm{~A}+2 \sin 5 \mathrm{~A}}=\frac{\sin 4 \mathrm{~A}}{\sin 5 \mathrm{~A}}$.

Solution : $\frac{\sin 6 \mathrm{~A}+\sin 2 \mathrm{~A}+2 \sin 4 \mathrm{~A}}{\sin 7 \mathrm{~A}+\sin 3 \mathrm{~A}+2 \sin 5 \mathrm{~A}}=\frac{2 \sin \left(\frac{8 \mathrm{~A}}{2}\right) \cdot \cos \left(\frac{4 \mathrm{~A}}{2}\right)+2 \sin 4 \mathrm{~A}}{2 \sin \left(\frac{10 \mathrm{~A}}{2}\right) \cdot \cos \left(\frac{4 \mathrm{~A}}{2}\right)+2 \sin 5 \mathrm{~A}}$
$=\frac{2 \sin 4 \mathrm{~A} \cos 2 \mathrm{~A}+2 \sin 4 \mathrm{~A}}{2 \sin 5 \mathrm{~A} \cos 2 \mathrm{~A}+2 \sin 50}=\frac{2 \sin 4 \mathrm{~A}(\cos 2 \mathrm{~A}+1)}{2 \sin 5 \mathrm{~A}(\cos 2 \mathrm{~A}+1)}=\frac{\sin 4 \mathrm{~A}}{\sin 5 \mathrm{~A}}$
42.Find the equation of the circle passing through the points $(1,-4),(5,2)$ and having its centre on the line $x-2 y+9=0$.
Solution : General equation of the circle is $x^{2}+y^{2}+2 g x+2 f y+c=0$

$$
\begin{align*}
& (1,-4) \rightarrow 2 g-8 f+c=-17 .  \tag{1}\\
& (5,2) \rightarrow 10 g+4 f+c=-29 . \tag{2}
\end{align*}
$$

Centre (-g,-f) on $\mathrm{x}-2 \mathrm{y}+9=0 \rightarrow-g+2 f=-9$ $\qquad$
Solving (1) and(2), $-2 \mathrm{~g}-3 \mathrm{f}=3$ (4)

Solving (3) and (4) $f=-3, g=3$ and $c=-47$
Then the equation of circle is $x^{2}+y^{2}+6 x-6 y-47=0$
43.Evaluate $\lim _{x \rightarrow 2}\left(\frac{x^{2}-4}{\sqrt{x+2}-\sqrt{3 x-2}}\right)$

$$
\begin{aligned}
& \text { Solution: } \lim _{x \rightarrow 2}\left(\frac{x^{2}-4}{\sqrt{x+2}-\sqrt{3 x-2}}\right) \times \lim _{x \rightarrow 2}\left(\frac{\sqrt{x+2}+\sqrt{3 x-2}}{\sqrt{x+2}+\sqrt{3 x-2}}\right) \\
& =\lim _{x \rightarrow 2}\left(\frac{(x+2)(x-2)}{\sqrt{x+2}-\sqrt{3 x-2}}\right) \times \lim _{x \rightarrow 2}\left(\frac{\sqrt{x+2}+\sqrt{3 x-2}}{\sqrt{x+2}+\sqrt{3 x-2}}\right) \\
& =\lim _{x \rightarrow 2}\left(\frac{(x+2)(x-2)}{(x+2)-(3 x-2)}\right) \times \lim _{x \rightarrow 2}(\sqrt{x+2}+\sqrt{3 x-2}) \\
& =\lim _{x \rightarrow 2}\left(\frac{(x+2)(x-2)}{-(x-2)}\right) \times \lim _{x \rightarrow 2}(\sqrt{x+2}+\sqrt{3 x-2}) \\
& =\left(\frac{(2+2)(2+2)}{-2}\right)=-8
\end{aligned}
$$

## PART-E

## VIII. Answer any TWO of the following questions :

44) A flag staff stands upon the top of a building at a distance of 20 mts . The angles of elevation of the top of the flag staff and the building are $60^{\circ}$ and $30^{\circ}$ respectively. Find the height of the flag staff .

## Solution:



Let the height of the flag staff $B C=h$
From triangle $\mathrm{BAD}, \tan 30^{\circ}=-$ then $\mathrm{AB}=\frac{-}{\sqrt{ }}$
From triangle $\mathrm{CAD}, \tan 60^{\circ}=-\quad$ then $\mathrm{h}+\mathrm{AB}=20 \sqrt{3} \quad \mathrm{~h}=\frac{\overline{\sqrt{3}}}{\sqrt{3}}=\frac{40 \sqrt{3}}{m}$
45)If $y=a \cos (\log x)+b \sin (\log x)$. Show that $x^{2} y_{2}+x y_{1}+y=0$

Solution: $\mathrm{y}=\mathrm{a} \cos (\log \mathrm{x})+\mathrm{b} \sin (\log \mathrm{x})$
Differentiating w.r.t x ,

$$
\begin{aligned}
& y_{1}=-\quad- \\
& x y_{1}=-a \sin (\log x)+b \cos (\log x)
\end{aligned}
$$

Again differentiating w.r.t $x$,

$$
\begin{aligned}
& x y_{2}+y_{1}= \\
& x^{2} y_{2}+x y_{1}+y=0
\end{aligned}
$$

46) The total revenue function is given by $R=400 x-2 x^{2}$ and the total cost function is given by $C=2 x^{2}+40 x+4000$. Find
a)The marginal revenue and marginal cost function
b)the output at which marginal revenue =marginal cost

Solution: Marginal revenue $=-$
Marginal cost=一

Output when $M R=M C$, i.e, $400-4 x=4 x+40$

$$
8 x=360
$$

## $X=45$ units

