DETAILED SOLUTIONS

1. In the expansion of $(1 + x)^n$ $\frac{C_1}{C_0}+2\frac{C_2}{C_1}+3\frac{C_3}{C_2}+...+n\frac{C_n}{C_{n-1}}$ is equal to

A) $\frac{n(n+1)}{2}$ B) $\frac{n}{2}$ C) $\frac{n+1}{2}$ D) 3n (n + 1)

Ans: A)

Given expression

$$=\frac{C_1}{C_0}+2\frac{C_2}{C_1}+3\frac{C_3}{C_2}+....+n\frac{C_n}{C_{n-1}}$$

$$=\frac{n}{1}+2\frac{n(n-1)}{2n}+....+n\left(\frac{1}{n}\right)$$

$$=n+(n-1)+(n-2)+....+2+1$$

$$=\frac{n(n+1)}{2}$$

2. If S_n stands for sum to n-terms of a G.P with 'a' as the first term and 'r' as the common ratio then $S_n : S_{2n}$ is

A) r^n+1 B) $\frac{1}{r^n+1}$ C) r^n-1 D) $\frac{1}{r^n-1}$

Ans: B)

$$\frac{S_n}{S_{2n}}=\frac{\frac{\not{a}(r^n-1)}{\not{r}-1}}{\frac{\not{a}(r^{2n}-1)}{\not{r}-1}}=\frac{(r^n-1)}{(r^{2n}-1)}=\frac{1}{r^n+1}$$

3. If A.M and G.M of roots of n quadratic equation are 5 and 4 respectively, then the quadratic equation is

A) $x^2-10x-16=0$ B) $x^2+10x+16=0$
C) $x^2+10x-16=0$ D) $x^2-10x+16=0$

Ans: D)

$$x^2-(a+b)x+(ab)=0$$

$$x^2-(2\times5)x+4^2=0$$

$$x^2-10x+16=0$$

4. The angle between the line x + y = 3 and the line joining the points (1, 1) and (- 3, 4) is

A) $\tan^{-1}(7)$ B) $\tan^{-1}\left(-\frac{1}{7}\right)$ C) $\tan^{-1}\left(\frac{1}{7}\right)$ D) $\tan^{-1}\left(\frac{2}{7}\right)$

Ans: C)

Angle = $\tan^{-1}\left|\frac{m_2-m_1}{1+m_1m_2}\right| = \tan^{-1}\left|\frac{-1+\frac{3}{4}}{1+\frac{3}{4}}\right| = \tan^{-1}\left|\frac{-1}{7}\right| = \tan^{-1}\frac{1}{7}$

5. The equation of parabola whose focus is (6, 0) and directrix is x = - 6 is

A) $y^2 = 24x$ B) $y^2 = -24x$ C) $x^2 = 24y$ D) $x^2 = -24y$

Ans: A)

Equation of parabola $y^2 = 4ax,\ here\ a = 6$

$\therefore y^2 = 24x$

6. $\lim_{x\to\frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1}$ is equal to

A) 2 B) $\sqrt{2}$ C) $\frac{1}{2}$ D) $\frac{1}{\sqrt{2}}$

Ans: C)

$$\lim_{x\to\frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1} = \lim_{x\to\frac{\pi}{4}}\left(\frac{\sqrt{2}(-\sin x) - 0}{-\operatorname{cosec}^2 x - 0}\right) = \left(\frac{\sqrt{2}\left(\frac{-1}{\sqrt{2}}\right)}{-\left(\sqrt{2}\right)^2}\right) = \frac{1}{2}$$

7. The negation of the statement "For every real number x; x^2 + 5 is positive" is

A) For every real number x; x^2 + 5 is not positive
B) For every real number x; x^2 + 5 is negative.
C) There exists at least one real number x such that x^2 + 5 is not positive.
D) There exists at least one real number x such that x^2 + 5 is positive.

Ans: C)

There exists atleast one real number x such that $x^2 + 5$ is not positive.

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8. Let a, b, c, d and e be the observations with mean m and standard deviation S. The standard deviation of the observations a + k, b + k, c + k, d + k is

A) kS B) S + K C) $\frac{S}{k}$ D) S

Ans: D)

Standard deviation remains same, if every observation is increased by a constant.

9. Let $f: R \to R$ be given by f(x) = tan x. Then f^{-1} (1) is

A) $\frac{\pi}{4}$ B) $\left\{n\pi + \frac{\pi}{4}; n \in Z\right\}$

C) $\frac{\pi}{3}$ D) $\left\{n\pi + \frac{\pi}{3}; n \in Z\right\}$

Ans: B)

$f_0 f^{-1}(x) = x$

$f_0 f^{-1}(1) = 1$

$f\left(f^{-1}(1)\right) = 1$

$\tan\left(f^{-1}(1)\right) = 1 = \tan\frac{\pi}{4}$

$f^{-1}(1) = n\pi + \frac{\pi}{4}$

10. Let f: R → R be defined by $f(x) = x^2 + 1$. Then the pre images of 17 and – 3 respectively are

A) $\phi, \{4, -4\}$ B) {3, - 3}, ϕ C) $\{4, -4\}, \phi$ D) {4, - 4}, {2, -2}

Ans: C)

$f(x) = x^2 + 1$. Pre images of 17 are 4 and – 4 which are roots of $x^2 + 1 = 17$.

But $x^2 + 1 = -3$ is not possible. Hence option C is correct.

11. Let $(gof)(x) = \sin x$ *and* $(fog)(x) = \left(\sin\sqrt{x}\right)^2$. Then

A) $f(x) = \sin^2 x, g(x) = x$ B) $f(x) = \sin\sqrt{x}, g(x) = \sqrt{x}$

C) $f(x) = \sin^2 x, g(x) = \sqrt{x}$ D) $f(x) = \sin\sqrt{x}, g(x) = x^2$

Ans: C)

$gof(x) = g(\sin^2 x) = \sqrt{\sin^2 x} = \sin x$

$fog(x) = f(\sqrt{x}) = \sin^2 \sqrt{x} = (\sin\sqrt{x})^2$

12. Let A={2,3,4,5,.....16,17,18}. Let R be the relation on the set A of ordered pairs of positive integers defined by $(a,b)R(c,d)$ if and only if $ad = bc$ for all $(a,b),(c,d)$ in $A \times A$. Then the number of ordered pairs of the equivalence class of (3,2) is

A) 4 B) 5 C) 6 D) 7

Ans: C)

$(3,2) \cong (x,y)$

3y = 2x

x = 3	y = 2
x = 6	y = 4
x = 9	y = 6
x = 12	y = 3
x = 15	y = 10
x = 18	y = 12

Total pairs = 6

13. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then $x(y+z)+y(z+x)+z(x+y)$ equals to

A) 0 B) 1 C) 6 D) 12

Ans: C)

$\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$

$\because 0 \le \cos^{-1} x \le \pi$

Max. of $\cos^{-1}(x) = \pi$

$\Rightarrow x = -1$, y = -1, z = -1

$\Rightarrow x(y+z)+y(z+x)+z(x+y)$

$= 2[xy + yz + zx]$

$= [1+1+1]$

=6

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14. If $2\sin^{-1}x-3\cos^{-1}x=4, x\in[-1,1]$ then $2\sin^{-1}x+3\cos^{-1}x$ is equal to

A) $\frac{4-6\pi}{5}$ B) $\frac{6\pi-4}{5}$ C) $\frac{3\pi}{2}$ D) 0

Ans: B)

$2\sin^{-1}x-3\cos^{-1}x=4$

$2\left(\frac{\pi}{2}-\cos^{-1}x\right)-3\cos^{-1}x=4$

$\pi-5\cos^{-1}x=4$

$\cos^{-1}x=\frac{\pi-4}{5}$

$\Rightarrow 3\cos^{-1}x=\frac{3\pi-12}{5} \quad ---(i)$

Similarly, $2\sin^{-1}x-3\left(\frac{\pi}{2}-\sin^{-1}x\right)=4$

$\sin^{-1}x=\left(\frac{4}{5}+\frac{3\pi}{10}\right)$

$2\sin^{-1}x=\left(\frac{8}{5}+\frac{3\pi}{5}\right) \quad ---(ii)$

From (i) and (ii)

$2\sin^{-1}x+3\cos^{-1}x=\frac{6\pi-4}{5}$

15. If A is a square matrix such that $A^2=A$, *then* $(I+A)^3$ is equal to

A) 7A-I B) 7A C) 7A+I D) I-7A

Ans: C)

$A^2=A$

$(I+A)^3=I^3+A^3+3I^2A+3IA^2=I+A+3A+3A=7A+I$

$\left(A^3=A^2.A=A.A^2=A^2=A\right)$

DETAILED SOLUTIONS

16. If $A=\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$, then A^{10} is equal to

A) $2^8 A$ B) $2^9 A$ C) $2^{10} A$ D) $2^{11} A$

Ans: B)

$A^2=\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}=2A$

$A^4=4A^2=8A$

$A^{10}=2^9 A$

17. If $f(x)=\begin{vmatrix}x-3 & 2x^2-18 & 2x^3-81\\ x-5 & 2x^2-50 & 4x^3-500\\ 1 & 2 & 3\end{vmatrix}$, then $f(1).f(3)+f(3).f(5)+f(5).f(1)$ is

A) -1 B) 0 C) 1 D) 2

Ans: Insufficient Data

If the question is like this, $f(x)=\begin{vmatrix}x-3 & 2x^2-18 & 2x^3-81\\ x-5 & 2x^2-50 & 4x^3-500\\ 1 & 2 & 3\end{vmatrix}$

Solution is given by:

$$f(x)=2(x-3)(x-5)\begin{vmatrix}1 & x+3 & 3(x^2+3x+9)\\ 1 & x+5 & 4(x^2+5x+25)\\ 1 & 1 & 3\end{vmatrix}$$

$f(3)=0, f(5)=0$

18. If $P=\begin{bmatrix}1 & \alpha & 3\\ 1 & 3 & 3\\ 2 & 4 & 4\end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A|=4$, then α is equal to

A) 4 B) 5 C) 11 D) 0

Ans: C)

$|adjA|=|A|^{3-1}=4^2=16$

$\Rightarrow |p| = 16$

$1(12-12) - \alpha(4-6) + 3(4-6) = 16$

$+22 - 6 = 16$

$2\alpha = 22$

$\alpha = 11$

19. If $A = \begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix}$ *and* $B = \begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$, then $\frac{dB}{dx}$ is

A) 3A   B) -3B   C) 3B+1   D) 1-3A

Ans: A)

$B = x(x^2 - 1) - 1(x - 1) + 1(1 - x)$

$B = x^3 - x - x + 1 + 1 - x$

$B = x^3 - 3x + 2$

Differentiate with respect to x

$\frac{dB}{dx} = 3x^2 - 3 = 3(x^2 - 1) = 3A \qquad \because |A| = x^2 - 1$

20. Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x & 2x \\ \sin x & x & x \end{vmatrix}$. Then $\lim_{x\to 0} \frac{f(x)}{x^2} =$

A) -1   B) 0   C) 3   D) 2

Ans: B)

$$\lim_{x\to 0} \frac{f(x)}{x^2} = \lim_{x\to 0} \begin{vmatrix} \cos x & x & 1 \\ \frac{2\sin x}{x} & 1 & 2 \\ \frac{\sin x}{x} & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{vmatrix} = 1[1-2] - 0(2-2) + 1[2-1] = -1 + 1 = 0$$

21. Which one of the following observations is correct for the features of logarithm function to any base $b > 1$?

A) The domain of the logarithm function is R, the set of real numbers.

B) The range of the logarithm function is R^+, the set of all positive real numbers.

C) The point (1,0) is always on the graph of the logarithm function

D) The graph of the logarithm function is decreasing as we move from left to right.

Ans: C)

$y=\log_b x (b>1)$

$(1,0)$ *satisfies*

22. The function $f(x)=|\cos x|$ is

A) Everywhere continuous and differentiable.

B) Everywhere continuous but not differentiable at odd multiples of $\frac{\pi}{2}$.

C) Neither continuous nor differentiable at $(2n+1)\frac{\pi}{2}, n \in Z$

D) Not differentiable everywhere.

Ans : B)

We know that |f(x)| is not differentiable when f(x) = 0 and composition of two continuous functions is also continuous.

23. If $y=2x^{3x}$, then $\frac{dy}{dx}$ at x = 1 is

A) 2 B) 6 C) 3 D) 1

Ans : B)

$\frac{dy}{dx}=2x^{3x}(3+3\log x)$. At x = 1, $\frac{dy}{dx}=6$

24. Let the function satisfy the equation f(x +y) = f(x) f(y) for all $x, y \in R$, where $f(0) \neq 0$. If f(5) =3 and f' (0) = 2, then f'(5) is

A) 6 B) 0 C) 5 D) -6

Ans : A)

$f(x)=a^x$

$f(5)=3 \Rightarrow a^5=3 \quad -----(1)$

$f'(x)=a^x \log a$

$f'(0)=\log a=2 \Rightarrow a=e^2$

$f'(5)=3.2=6$

25. The value of C in (0,2) satisfying the mean value theorem for the function $f(x)=x(x-1)^2$, $x \in [0,2]$ is equal to

A) $\frac{3}{4}$ B) $\frac{4}{3}$ C) $\frac{1}{3}$ D) $\frac{2}{3}$

Ans : B)

$f(x) = x(x-1)^2 = x(x^2+1-2x) = x^3 - 2x^2 + x$

f is continuous on [0, 2] and differentiable on [0, 2]

$f^1(x) = 3x^2 - 4x + 1$

$f(0) = 0 \qquad f(2) = 2$

$\therefore f^1(c) = \dfrac{f(b)-f(a)}{b-a}$

$3c^2 - 4c + 1 = \dfrac{2-0}{2-0}$

$3c^2 - 4c + 1 = 1$

$3c^2 = 4c$

$3c = 4$

$c = \dfrac{4}{3} \in (0, 2)$

26. $\dfrac{d}{dx}\left[\cos^2\left(\cot^{-1}\sqrt{\dfrac{2+x}{2-x}}\right)\right]$ is

A) $-\dfrac{3}{4}$ B) $-\dfrac{1}{2}$ C) $\dfrac{1}{2}$ D) $\dfrac{1}{4}$

Ans : D)

$\dfrac{dx}{d\theta} = 2(2)(-\sin 2\theta)$

$= -4\sin 2\theta$

$\dfrac{d}{dx}\left[\cos^2\left(\cot^{-1}\sqrt{\dfrac{2+x}{2-x}}\right)\right]$

$= \dfrac{d}{dx}\left[\cos^2\left(\tan^{-1}\sqrt{\dfrac{2-x}{2+x}}\right)\right], \quad x = 2\cos 2\theta$

$= \dfrac{d}{dx}\left[\cos^2\left(\tan^{-1}\sqrt{\dfrac{2(1-\cos 2\theta)}{2(1+\cos 2\theta)}}\right)\right] = \dfrac{d}{dx}\left[\cos^2\left(\tan^{-1}\sqrt{\dfrac{2\sin 2\theta)}{2\cos 2\theta)}}\right)\right]$

$= \dfrac{d}{dx}\left[\cos^2(\tan^{-1}(\tan\theta))\right] = \dfrac{d}{dx}(\cos^2\theta) = 2\cos\theta \times -\sin.\theta \dfrac{d\theta}{dx} = \dfrac{-\sin 2\theta}{\left(\dfrac{dx}{d\theta}\right)}$

$= \dfrac{-\sin 2\theta}{-4\sin 2\theta} = \dfrac{1}{4}$

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27. For the function $f(x) = x^3 - 6x^2 + 12x - 3;\ x = 2$ is

A) A point of minimum
B) a point of inflexion
C) Not a critical point
D) a point of maximum

Ans : B)

$$f(x) = x^3 - 6x^2 + 12x - 3$$
$$f'(x) = 3x^2 - 12x + 12$$
$$f''(x) = 6x - 12 = 0 \Rightarrow x = 2$$
$$f''(2^+) > 0 \ \textit{and}\ f''(2^-) < 0$$

$\because$ Since sign of concavity changes about x = 2

$\Rightarrow$ x = 2 is a point of inflection

28. The function $x^x; x > 0$ is strictly increasing at

A) $\forall x \in \mathbb{R}$
B) $x < \frac{1}{e}$
C) $x > \frac{1}{e}$
D) x<0

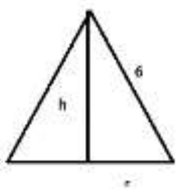
Ans : C)

$$y = x^x$$
$$\log y = x \log x \Rightarrow \frac{1}{y}\frac{dy}{dx} = \log x + 1$$
$$or\ \frac{dy}{dx} = x^x(\log x + 1)$$
$$x^x > 0 \Rightarrow \log x + 1 > 0$$
$$or\ \log x > -1\ \ or\ \ x > e^{-1}$$

29. The maximum volume of the right circular cone with slant height 6 units is

A) $4\sqrt{3}\pi$ cubic units
B) $16\sqrt{3}\pi$ cubic units
C) $3\sqrt{3}\pi$ cubic units
D) $6\sqrt{3}\pi$ cubic units

Ans : B)



v = $\frac{1}{3}\pi r^2 h$

$r^2 = (36 - h^2)$

$v = \frac{\pi}{3}(36 - h^2).h$

v = $12\pi h - \frac{\pi h^3}{3}$

$\frac{dv}{dh} = 12\pi - \left(\frac{\pi}{3}\right)(3h^2) = 12\pi - \pi h^2$

$= \pi(12 - h^2) = 0$

$\Rightarrow h^2 = 12 \Rightarrow h = 2\sqrt{3}$

$r^2 = 36 - 12 \; or \; r^2 = 24$

$v = 12\pi h - \frac{\pi h^3}{3}$

$v_{\max} = \frac{1}{3} \times \pi \times 24 \times 2\sqrt{3}$

$v_{\max} = 16\sqrt{3}\pi$ cubic units

30. If $f(x) = xe^{x(1-x)}$ then f(x) is

A) Increasing in $\mathbb{R}$

B) decreasing $\mathbb{R}$

C) Decreasing in $\left[-\frac{1}{2}, 1\right]$

D) Increasing in $\left[-\frac{1}{2}, 1\right]$

Ans : D)

$f(x) = xe^{x(1-x)}$

$f'(x) = e^{x(1-x)} + e^{x(1-x)}.(1-2x).x = e^{x(1-x)}\left[1 + x - 2x^2\right] \geq 0$

$e^{x(1-x)} > 0 \Rightarrow 1 + x - 2x^2 \geq 0$

$\Rightarrow 2x^2 - x - 1 \leq 0 \; or \; (2x+1)(x-1) \leq 0$

$or \; x \in \left[\frac{-1}{2}, 1\right] \; increasing \; in \left[\frac{-1}{2}, 1\right]$

31. $\int \frac{\sin x}{3 + 4\cos^2 x} dx =$

A) $-\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right) + c$

B) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\cos x}{\sqrt{3}}\right) + c$

C) $\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{\cos x}{\sqrt{3}}\right) + c$

D) $-\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right) + c$

Ans : A)

$$\int \frac{\sin x\ dx}{\left(\sqrt{3}\right)^2 + (2\cos x)^2}$$

$2\cos x = t$

$-2\sin x\ dx = dt$

$$= -\frac{1}{2}\int \frac{dt}{\left(\sqrt{3}\right)^2 + t^2} \quad = -\frac{1}{3} \times \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t}{\sqrt{3}}\right)$$

$$-\frac{1}{2\sqrt{3}} \tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right) + C$$

32. $\int_{-\pi}^{\pi} \left(1-x^2\right)\sin x.\cos^2 x\ dx =$

A) $\pi - \frac{\pi^2}{3}$ B) $2\pi - \pi^3$ C) $\pi - \frac{\pi^3}{2}$ D) 0

Ans : D)

$$\int_{\pi}^{\pi} \left(1-x^2\right)\sin x\ \cos^2\ xdx$$

t(x) = $\left(1-x^2\right)\ \sin x\ \cos^2 x$

$t(-x) = \left(1-x^2\right)\sin(-x)\cos^2 x = -t(x)$

$\therefore t(x)$ is odd function

$$\int_{\pi}^{\pi} \left(1-x^2\right)\sin x\ \cos^2\ xdx = 0$$

33. $\int \frac{1}{\mathrm{x}[6(\log \mathrm{x})^2 + 7\log \mathrm{x} + 2} \mathrm{dx} =$

A) $\frac{1}{2}\log\left|\frac{2\ \log\ x+1}{3\ \log\ x+2}\right| + C$ B) $\log\left|\frac{2\ \log\ x+1}{3\ \log\ x+2}\right| + C$

C) $\log\left|\frac{3\ \log\ x+2}{2\ \log\ x+1}\right| + C$ D) $\frac{1}{2}\log\left|\frac{3\ \log\ x+2}{2\ \log\ x+1}\right| + C$

Ans : B)

$\log x = t \Rightarrow \frac{1}{x}dx = dt$

$$\int \frac{dt}{6t^2+7t+2} = \int \frac{dt}{(3t+2)(2t+1)}$$

$$= \int \left(\frac{-3}{3t+2} + \frac{2}{2t+1}\right) dt = \frac{-3\log(3t+2)}{3} + \frac{2\log(2t+1)}{2} + C = \log\left(\frac{2\log x+1}{3\log x+2}\right) + C$$

34. $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}} dx =$

A) 2x + sin x + 2 sin 2x + C
B) x + 2 sin x + 2 sin 2x + C
C) x + 2 sin x + sin 2x + C
D) 2x + sin x + sin 2x + C

Ans : C)

$$\int \frac{2\sin\frac{5x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}} dx$$

$$\int \frac{\sin 3x + \sin 2x}{\sin x}$$

$$\int \left(3 - 4\sin^2 x + 2\cos x\right) dx$$

$$\int \left(3 - 2 + 2\cos 2x + 2\cos x\right) dx$$

$$= x + 2\sin x + \sin 2x + c$$

35. $\int_1^5 (|x-3| + |1-x|)\, dx =$

A) 12
B) $\frac{5}{6}$
C) 21
D) 10

Ans : A)

$$\int_1^3 (3 - x + x - 1) dx + \int_3^5 (x - 3 + x - 1) dx$$

$$= 2(x)_1^3 + \left(x^2 - 4x\right)_3^5 = 2(2) + \left((25-20) - (9-12)\right) = 4 + 5 + 3 = 12$$

36. $\lim_{n\to\infty}\left(\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \frac{n}{n^2+3^2} + ... + \frac{1}{5n}\right) =$

A) $\frac{\pi}{4}$
B) $\tan^{-1} 3$
C) $\tan^{-1} 2$
D) $\frac{\pi}{2}$

Ans : C)

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$$\lim_{n\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\dots+\frac{1}{5n}\right)$$

$$=\lim_{n\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\dots+\frac{n}{n^2+(2n)^2}\right)$$

$$=\lim_{n\to\infty}\sum_{r=1}^{2n}\frac{n}{n^2+r^2}$$

$$\lim_{n\to\infty}\sum_{r=1}^{2n}\frac{1}{n}\left(\frac{1}{1+\left(\frac{r}{n}\right)^2}\right)=\int_0^2\frac{dx}{1+x^2}=\tan^{-1}(2)$$

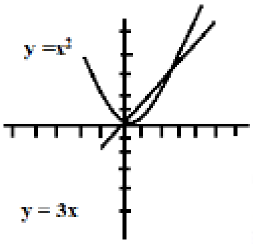
37. The area of the region bounded by the line y = 3x and the curve y = x^2 in sq. units is

A) 10 B) $\frac{9}{2}$ C) 9 D) 5

Ans : B)

$x^2=3x\Rightarrow x^2-3x=0\Rightarrow x(x-3)=0\Rightarrow x=0,3$

$$\int_0^3 3x-\int_0^3 x^2=\frac{3x^2}{2}\Big|_0^3-\frac{x^3}{3}\Big|_0^3=3\left(\frac{9}{2}\right)-\frac{27}{3}=\frac{27}{2}-\frac{27}{3}=\frac{81-54}{6}=\frac{27}{6}=\frac{9}{2}$$



38. The area of the region bounded by the line y = x and the curve y = x^3 is

A) 0.2 sq. units B) 0.3 sq. units C) 0.4 sq. units D) 0.5 sq. units

Ans : D)

$y=x^3$ and $y=x\Rightarrow x^3=x\Rightarrow x^3-x=0\Rightarrow x(x^2-1)=0\Rightarrow x=0,x=\pm1$

$$A_1=\int_0^1(x-x^3)dx=\frac{x^2}{2}-\frac{x^4}{4}\Big|_0^1=\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$$

$$A_2=\int_{-1}^0 x^3-x=\int_0^{-1}x-x^3=\frac{x^2}{2}-\frac{x^4}{4}\Big|_0^1=\frac{1}{2}-\frac{1}{4}=\frac{-1}{4}$$

39. The solution of $e^{\frac{dy}{dx}}=x+1,\ y(0)=3$ is

A) y – 2 = x log x – x B) y – x – 3 = x log x

C) y – x – 3 = (x + 1) log (x + 1) D) y + x – 3 = (x + 1) log (x + 1)

Ans : D)

$e^{\frac{dy}{dx}} = x + 1$

$\frac{dy}{dx} = \log(x+1)$

$\int dy = \int \log(x+1)\,dx$

Integrate with respect to x

y = x log (x + 1) – x + log (x + 1) + C

Put x = 0, y = 3

3 = 0 log (1) – 0 + log (1) + C

⟹ C = 3

y = x log (x + 1) – x + log (x + 1) + 3

y + x – 3 = x log (x + 1) + log (x + 1)

y + x – 3 = log (x + 1) {x + 1}

40. The family of curves whose x and y intercepts of a tangent at any point are respectively double the x and y coordinates of that point is

A) xy = C B) $x^2 + y^2 = C$ C) $x^2 - y^2 = C$ D) $\frac{y}{x} = C$

Ans : A)

Let point be (h, k)

Equation of tangent $\frac{x}{2h} + \frac{y}{2k} = 1$

$y = \frac{-2k}{2h}x + 2k$

$\text{slope} = \frac{-k}{h}$

Replace (h, k) by (x, y)

$\frac{dy}{dx} = \frac{-y}{x} \Rightarrow \frac{dy}{y} = \frac{-dx}{x}$

Integrate

log y = – log x + log C

⟹ xy = C

41. The vectors $\overrightarrow{AB} = 3\hat{i} + 4k$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4k$ are the sides of a Δ ABC. The length of the median through A is

A) $\sqrt{18}$ B) $\sqrt{72}$ C) $\sqrt{33}$ D) $\sqrt{288}$

Ans : C)

Median = $\frac{\overrightarrow{AB}+\overrightarrow{AC}}{2} = 4\hat{i} - j + 4k$

Length of median = $\sqrt{16+1+16} = \sqrt{33}$

42. The volume of the parallelopiped whose co-terminous edges are $\hat{j}+k$, $\hat{i}+k$ and $\hat{i}+\hat{j}$ is

A) 6 cu. units B) 2 cu. units C) 4 cu. units D) 3 cu. units

Ans : B)

Volume of parallelepiped = $\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$ = 0-1(0-1)+1(1)

= 2 cu. Unit

43. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a} + \vec{b}$ is a unit vector if

A) $\theta = \frac{\pi}{4}$ B) $\theta = \frac{\pi}{3}$ C) $\theta = \frac{2\pi}{3}$ D) $\theta = \frac{\pi}{2}$

Ans: C)

$|\vec{a}| = 1 = |\vec{b}| = |\vec{a}+\vec{b}|$

W.k.t $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow 1 = 1+1+2\cos\theta$

$\Rightarrow \cos\theta = \frac{-1}{2} \Rightarrow \theta = \frac{2\pi}{3}$

44. If $\vec{a}, \vec{b}, \vec{c}$ are three coplanar vectors and p, q, r vectors defined by

$\vec{p} = \frac{\vec{b}\times\vec{c}}{[\vec{a}\vec{b}\vec{c}]}, \vec{q} = \frac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]}, \vec{r} = \frac{\vec{a}\times\vec{b}}{[\vec{a}\vec{b}\vec{c}]}$, then $(\vec{a}+\vec{b}).\vec{p} + (\vec{b}+\vec{c}).\vec{q} + (\vec{c}+\vec{a}).\vec{r}$ is

A) 0 B) 1 C)2 D) 3

Ans: D)

$$(\vec{a}+\vec{b}).\vec{p} + (\vec{b}+\vec{c}).\vec{q} + (\vec{c}+\vec{a}).\vec{r} = (\vec{a}+\vec{b}).\frac{(\vec{b}\times\vec{c})}{[\vec{a}\vec{b}\vec{c}]} + (\vec{b}+\vec{c}).\frac{(\vec{c}\times\vec{a})}{[\vec{a}\vec{b}\vec{c}]} + (\vec{c}+\vec{a}).\frac{(\vec{a}\times\vec{b})}{[\vec{a}\vec{b}\vec{c}]}$$

$$= \frac{\vec{a}.(\vec{b}\times\vec{c})}{[\vec{a}\vec{b}\vec{c}]} + 0 + \frac{\vec{b}.(\vec{c}\times\vec{a})}{[\vec{a}\vec{b}\vec{c}]} + 0 + \frac{\vec{c}.(\vec{a}\times\vec{b})}{[\vec{a}\vec{b}\vec{c}]} + 0 = 1+1+1 = 3$$

DETAILED SOLUTIONS

45. If lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ *and* $\frac{x-1}{3k}=\frac{y-5}{1}=\frac{z-6}{-5}$ are mutually perpendicular then k is equal to

A) $-\frac{10}{7}$ B) $-\frac{7}{10}$ C) – 10 D) – 7

Ans: A)

Given $a_1,b_1,c_1=-3,2k,2$ and $a_2,b_2,c_2=3k,1,-5$

$$a_1a_2+b_1b_2+c_1c_2=0 \Rightarrow -9k+2k-10=0 \Rightarrow -7k=10 \Rightarrow k=\frac{-10}{7}$$

46. The distance between the two planes 2x +3y+ 4z = 4 and 4x+ 6y+8z = 12 is

A) 2 units B) 8 units C) $\frac{2}{\sqrt{29}}$ *units* D) 4 units

Ans: C)

$$2x+3y+4z=4$$
$$2x+3y+4z=6$$
$$d=\frac{|6-4|}{\sqrt{4+9+16}}=\frac{2}{\sqrt{29}}$$

47. The sine of the angle between the straight line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{4-z}{-5}$ and the plane 2x – 2y + z = 5 is

A) $\frac{1}{5\sqrt{2}}$ B) $\frac{2}{5\sqrt{2}}$ C) $\frac{3}{50}$ D) $\frac{3}{\sqrt{50}}$.

Ans: A)

$$\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$$

This line is parallel to the vector $\vec{b}=3\hat{i}+4j+5k$

Equation of plane is 2x – 2y + z = 5

Normal to the plane is $\vec{n}=2\hat{i}-2j+k$

We know that, $\sin\theta=\left|\frac{\vec{b}.\vec{n}}{|\vec{b}||\vec{n}|}\right|=\frac{1}{5\sqrt{2}}$

DETAILED SOLUTIONS

48. The equation xy = 0 in three dimensional space represents
   A) A pair of straight lines
   B) A plane
   C) A pair of planes at right angles
   D) A pair of parallel panes

   Ans: C)

   If xy = 0 $\Rightarrow x=0 \quad y=0$, i.e x = 0 $\rightarrow$ YZ-plane
   $y=0 \rightarrow XZ$-plane
   These two are planes at right angles.

49. The plane containing the point (3, 2, 0) and the line $\frac{x-3}{1}=\frac{y-6}{5}=\frac{z-4}{4}$ is
   A) $x-y+z=1$
   B) $x+y+z=5$
   C) $x+2y-z=1$
   D) $2x-y+z=5$

   Ans: A)

   Satisfies (3, 2, 0) and $a_1a_2+b_1b_2+c_1c_2=0 \Rightarrow 1(1)-1(5)+1(4)=0$

50. Corner points of the feasible region for an LPP are (0,2), (3,0), (6,0), (6,8) and (0,5). Let $z=4x+6y$ be the objective function. The minimum value of z occurs at
   A) Only (0,2)
   B) only (3,0)
   C) The mid-point of the line segment joining the points (0,2) and (3,0)
   D) Any point on the line segment joining the points (0,2) and (3,0)

   Ans: D)

   $Z_{(0,2)}=12 \leftarrow$ min
   $Z_{(3,0)}=12 \leftarrow$ min
   $Z_{(6,0)}=24$
   $Z_{(6,8)}=24+48=72$
   $Z_{(0,5)}=30$
   Any point on the line segment joining the point (0,2) and (3,0).

51. A die thrown 10 times. The probability that an odd number will come up at least once is
   A) $\frac{11}{1024}$
   B) $\frac{1013}{1024}$
   C) $\frac{1023}{1024}$
   D) $\frac{1}{1024}$

Ans: C)

$n=10,\ p=\frac{1}{2},\ q=\frac{1}{2}$

$P(x\ge 1)=1-P(x=0)=1-\frac{1}{2^{10}}=\frac{1023}{1024}$

52. A random variable X has the following probability distribution:

X	0	1	2
P(X)	$\frac{25}{36}$	k	$\frac{1}{36}$

If the mean of the random variable X is $\frac{1}{3}$, then the variance is

A) $\frac{1}{18}$ B) $\frac{5}{18}$ C) $\frac{7}{18}$ D) $\frac{11}{18}$

Ans: B)

$\sum p(x)=1$

$\frac{25}{36}+\frac{1}{36}+k=1$

$k=\frac{10}{36}$

$V(x)=E(x^2)-[E(x)]^2=\left(\frac{10}{36}+\frac{4}{36}\right)-\frac{1}{9}=\frac{5}{18}$

53. If a random variable X follows the binomial distribution with parameters n=5, p and $P(X=2)=9P(X=3)$, then p is equal to

A) 10 B) $\frac{1}{10}$ C) 5 D) $\frac{1}{5}$

Ans: B)

$P(x=2)=9P(x=3)$

${}^5C_2 p^2(1-p)^3=9\,{}^5C_3 p^3(1-p)^2 \Rightarrow \frac{1-p}{p}=9 \Rightarrow p=\frac{1}{10}$

54. Two finite sets have m and n elements respectively. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n respectively are

A) 7,6 B) 5,1 C) 6,3 D) 8,7

Ans: C)

$2^m - 2^n = 56 \quad ----(i)$

$2^n\left(2^{m-n} - 1\right) = 8 \times 7 = 2^3\left(2^3 - 1\right)$

$\therefore n = 3 \text{ and } m - n = 3 \Rightarrow m = 6$

55. If $[x]^2 - 5[x] + 6 = 0$, where $[x]$ denotes the greatest integer function, then

A) $x \in [3,4]$ B) $x \in [2,4)$ C) $x \in [2,3]$ D) $x \in (2,3]$

Ans: B)

$[x]^2 - 5[x] + 6 = 0 \qquad [x] = 2,3 \Rightarrow x \in [2,4)$

56. If in two circles, arcs of the same length subtend angles 30° and 78° at the centre, then the ratio of their radii is

A) $\frac{5}{13}$ B) $\frac{13}{5}$ C) $\frac{13}{4}$ D) $\frac{4}{13}$

Ans: B)

$\theta_1 = 30 = \frac{\pi}{6}, \theta_2 = 78 = \frac{\pi}{180} \times 78 = \frac{13\pi}{30}$

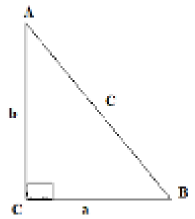
$WKT \ \ \theta_1 r_1 = \theta_1 r_2$

$\frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{13\frac{\pi}{30}}{\frac{\pi}{6}} = \frac{13 \times 6}{30_5} = \frac{13}{5}$

57. If ΔABC is right angled at C, then the value of tan A+ tan B is

A) a + b B) $\frac{a^2}{bc}$ C) $\frac{c^2}{ab}$ D) $\frac{b^2}{ac}$

Ans: C)



$\tan A=\frac{a}{b}, \tan B=\frac{b}{a}$

$\tan A+\tan B=\frac{a}{b}+\frac{b}{a}=\frac{a^2+b^2}{ab}=\frac{c^2}{ab}$

58. The real value of 'α' for which $\frac{1-i\sin\alpha}{1+2i\sin\alpha}$ is purely real is

A) $(n+1)\frac{\pi}{2}, n\in N$

B) $(2n+1)\frac{\pi}{2}, n\in N$

C) $n\pi, n\in N$

D) $(2n-1)\frac{\pi}{2}, n\in N$

Ans: C)

$z=\frac{a+ib}{c+id}\Rightarrow \text{Im}(z)=\frac{bc-ad}{c^2+d^2}$

$z=\frac{1-i\sin\alpha}{1+2i\sin\alpha}$

$\text{Im}(z)=0$

$\frac{-\sin\alpha-2\sin\alpha}{1+4\sin^2\alpha}=0$

$-3\sin\alpha=0$

$\sin\alpha=0$

$\alpha=n\pi, n\in N$

59. The length of a rectangle is five times the breadth. If the minimum perimeter of the rectangle is 180 cm, then

A) Breadth ≤ 15 cm

B) Breadth ≥ 15 cm

C) Length ≤ 15 cm

D) Length $= 15$ cm

Ans: B)

$l = 5b$

$2(l + b) \geq 180$

$l + b \geq 90$

$5b + b \geq 90$

$6b \geq 90$

$b \geq 15$ cm

60. The value of $^{49}C_3 + ^{48}C_3 + ^{47}C_3 + ^{46}C_3 + ^{45}C_3 + ^{45}C_4$ is

A) $^{50}C_4$ B) $^{50}C_3$ C) $^{50}C_2$ D) $^{50}C_1$

Ans: A)

We know that $^{n}C_r + ^{n}C_{r-1} = ^{n+1}C_r$

From back of the series,

$$^{45}C_4 + ^{45}C_3 = ^{46}C_4$$
$$^{46}C_4 + ^{46}C_3 = ^{47}C_4$$
$$^{47}C_4 + ^{47}C_3 = ^{48}C_4$$
$$^{48}C_4 + ^{48}C_3 = ^{49}C_4$$
$$^{49}C_4 + ^{49}C_3 = ^{50}C_4$$

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